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# Tracking Reference Orbits Around Asteroids with Unknown Gravitational Parameters Using a Nonlinear Adaptive Controller

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**In this paper, a nonlinear adaptive controller is implemented to track reference orbit trajectories around asteroids or small bodies. The two-body problem with a rotating reference frame is used to model the dynamics of the spacecraft with respect to the asteroid frame of reference. Gravitational perturbations, solar radiation pressure and other unmodeled dynamics are taken as disturbances to the system model. This type of controller has several advantages over other conventional controllers in the presence of uncertain system properties. Preliminary results shows accurate tracking of desired trajectories around asteroid Ida in the presence of unknown gravitational perturbations.**

## I. Nomenclature

$r$	=	position vector
$\dot{r}$	=	velocity vector
$\Omega$	=	angular velocity vector
$u$	=	control effort vector
$u_D$	=	gravitational perturbations
$e_y$	=	output error
$K_e(t)$	=	adaptive gain corresponding to output error
$K_r(t)$	=	adaptive gain corresponding to reference state
$\sigma_e$ and $\sigma_r$	=	adaptive gain tuning parameters
$U$	=	gravitational potential energy
$\omega$	=	magnitude of angular velocity

## II. Introduction

There has been a growing interest in the near earth asteroids. Some applications include exploring them for scientific purposes, earth-asteroid collision prevention and even for future asteroid mining for natural resources. However, due to their asymmetric shapes, non-uniform density, weak gravitational field and other time varying parameters, like rotation states etc, it is difficult to precisely model the spacecraft dynamics around them. Currently, asteroid missions take several years of research to estimate these parameters from earth. Even for close proximity missions like Hayabusa 1 and 2, it take months to study the nature of asteroids and these missions require constant ground control which results in error and dependence on ground stations. There is also a possibility of using an adaptive controller during component failure in spacecraft. For example, during the Hayabusa 1 mission, there was a control thrust failure during one of the test landing. An adaptive control law could account for these failures in future missions saving significant costs and manual input.

A simple adaptive control law algorithm [1] is applied in this work. The advantage of this control law is that a high fidelity system model is not required for trajectory tracking. Space-based applications are often faced with challenges where accurate data is not available due to various reasons. For example, rotational rate and moment of inertia of small asteroids are not very accurate due to large distances from earth, cost of equipment and manpower.

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There have been several new developments in the area of adaptive control for asteroid missions. Lee and Singh [2] discusses a novel adaptive control strategy to track trajectories using immersion and in-variance based adaptive control. Many developments are made in particular for hovering cases near an asteroid. Lee, Sanyal, Butcher and Scheeres [3] proposed a control method in geometric mechanics framework. This type of framework is more robust to coupling effects between orbit and attitude dynamics. Another paper by Nazari [4] proposed an optimal control approach for the hovering problem. Here gravitational parameters are first estimated using an extended Kalman filter (EKF) and then two types of optimal control strategies are then discussed.

In this paper, a direct adaptive control law is used to stabilize the spacecraft at desired orbit around a non-uniform rotating celestial body. This type of control is also known as a model free (autonomous) controller. Which means that no knowledge of the system is required to control the system. In this case, asteroid and spacecraft parameters are not required to implement the control. Only the measurements of tracked states and reference states are required. This makes the control law very robust for implementation. The downside of this type of direct adaptive controller is the loss of optimality in terms of control effort. For the purposes of modeling the spacecraft, only the two-body dynamics in the rotating body frame is considered. Perturbations due to the non-uniform gravitational field are taken as a disturbance to the system model and the adaptive control law is able to achieve the desired stable orbit without any prior knowledge about these disturbances. In the future, it is possible to include solar radiation pressure, tumbling and other disturbances like torques due to out-gassing. Shi [5] used similar adaptive control algorithm for attitude control in spacecraft. An application of spacecraft docking can be found in Prabhakar and Tiwari [6]. Again, a direct adaptive controller (DAC) is used for docking two spacecrafts in the presence of unknown gravitational perturbations.

### III. Dynamics of relative motion

The dynamics of the spacecraft in rotating body frame of reference for a uniformly rotating asteroid can be given as,

$$\ddot{r} + 2\Omega \times \dot{r} + \Omega \times (\Omega \times r) = \frac{\partial U}{\partial r} + a_{srp} + F + u \quad (1)$$

Where,  $u$  is the thrust control vector,  $a_{SRP}$  is the acceleration due to solar radiation pressure,  $F$  is the disturbance vector due to other unmodeled disturbances and  $\frac{\partial U}{\partial r}$  is acceleration due to the gravity of the asteroid. Also,  $u, r, \Omega, u, a_{SRP} \in R^3$  and  $F \in R^3$ .

#### A. Forces

##### 1. Modeling Gravity using MacCullagh's Approximation

MacCullagh's Approximation retains only first 3 terms of the infinite series expression of the gravitational potential field [7].

The gravitational field of an arbitrary body with a body-fixed frame at the center of mass is given as

$$V(r) = -\frac{Gm}{|r|} - \frac{G}{2|r|^3}(I_A + I_B + I_C - 3I_r) \quad (2)$$

Here, A,B,C are the moment of inertia of the asteroid with respect body fixed coordinate axes  $x, y$  and  $z$ . It can be noted here that for a spherical body  $I_A = I_B = I_C = I_r$  which equals to the point mass modeling of the gravity field for a spherical body [8].

$$I_r = \frac{1}{|r|^2}(I_A x^2 + I_B y^2 + I_C z^2) \quad (3)$$

The gravitational acceleration is given as

$$\frac{\partial U}{\partial r} = -\frac{\mu}{|r|^3}r - \frac{3G}{2|r|^5}(I_A + I_B + I_C - 5I_r)r - \frac{3G}{|r|^5}(I_A x\hat{x} + I_B y\hat{y} + I_C z\hat{z}) \quad (4)$$

Gravity acceleration in  $a_x, a_y, a_z$  components are given by

$$a_x = -\frac{\mu}{|r|^3}x - \frac{3}{2}G(I_A + I_B + I_C - 5I_r) + 3GI_A \quad (5)$$

$$a_y = -\frac{\mu}{|r|^3}y - \frac{3}{2}G(I_A + I_B + I_C - 5I_r) + 3GI_B \quad (6)$$

$$a_z = -\frac{\mu}{|r|^3}z - \frac{3}{2}G(I_A + I_B + I_C - 5I_r) + 3GI_C \quad (7)$$

## 2. Solar Radiation Pressure (SRP) Model

To model acceleration due to solar radiation pressure, a flat plate model is adopted. This model is used in [9] to model Hayabusa2's SRP model. Without the loss of generality, the plate is assumed to be perpendicular to  $z^{SC}$  axis. For getting maximum power output from the solar array the goal would be to align  $z^{SC}$  axis with the sun point  $z_{SP}$ . Therefore, ideally  $\hat{z}^{SC} \cdot \hat{z}^{SP} = 1$ . Here,  $\hat{z}^{SC}$  and  $\hat{z}^{SP}$  are axes aligned with sun pointing axis and axis perpendicular to the solar array respectively. Also, to maximize the effect of SRP,  $\hat{z}^{SC} \cdot \hat{z}^{SP} = 1$ . For a flat plate model, SRP is given by

$$a_{SRP} = -\frac{PA}{m}(\hat{z}^{SC} \cdot \hat{z}^{SP})[2((\hat{z}^{SC} \cdot \hat{z}^{SP})C_s + B_f C_d)n + (C_d + C_a)\hat{z}^{SP}] \quad (8)$$

Where,  $B_f = \frac{2}{3}$  is the Lambertian coefficient;  $A$  is the surface area of the flat plate; and  $P = \frac{P_0}{d^2}$  is the SRP acting on the surface of the spacecraft. Also,  $P_0 = 1 \times 10^{17} \frac{kg \cdot m}{s^2}$  and  $C_s, C_d, C_a$  corresponds to optical constant of the spacecraft. A much simpler model as shown below can be considered as well,

$$a_{SRP} = -a_0 \cos^2(\theta) \hat{n} \quad (9)$$

Where  $\cos(\theta)$  is the dot product between  $\hat{z}^{SC}$  and  $\hat{z}^{SP}$ , which can be taken as 1 to maximize the disturbance acceleration caused by SRP. Other forms sinusoidal of disturbances can also be used similar to Shi [5].

## B. Component wise dynamics

For the purposes of this research and without the loss of generality the angular velocity vector of the asteroid is aligned with the  $z$  axis of the asteroid. In future, tumbling cases will also be considered where the body fixed angular velocity vector is periodic. For completeness, the equations in Cartesian frame is expressed in (10)-(12),

$$\ddot{x} - 2\omega\dot{y} - \omega^2x = a_x + a_{SRP_x} + F_{dist_x} + u_x \quad (10)$$

$$\ddot{y} + 2\omega\dot{x} - \omega^2y = a_y + a_{SRP_y} + F_{dist_y} + u_y \quad (11)$$

$$\ddot{z} = a_z + a_{SRP_z} + F_{dist_z} + u_z \quad (12)$$

Where  $\omega$  is the magnitude of the angular velocity of the asteroid in in the  $z$  axis.

## IV. Control Structure

The form of an direct adaptive controller is followed from [1]

$$u = K_e(t)e_y + K_x(t)x_m + K_u(t)u_m \quad (13)$$

Where  $K_e(t) \in R^{m \times m}$  is the time-varying control gain matrix,  $K_x(t) \in R^{m \times n}$  and  $K_u(t) \in R^{m \times m}$ . Also, each control gain matrix is a summation of integral and proportional gains,

$$K_e(t) = K_{Ie}(t) + K_{Pe}(t) \quad (14)$$

$$K_x(t) = K_{Ix}(t) + K_{Px}(t) \quad (15)$$

$$K_u(t) = K_{Iu}(t) + K_{Pu}(t) \quad (16)$$

The integral control update law is given as,

$$\dot{K}_{Ie}(t) = -e_y(t)e_y^T(t)\Gamma_e \quad (17)$$

$$\dot{K}_{Ix}(t) = -e_y(t)x_{ref}^T\Gamma_r \quad (18)$$

$$\dot{K}_{Iu}(t) = -e_y(t)u^T\Gamma_u \quad (19)$$

$$\Gamma_e, \Gamma_r, \Gamma_u > 0 \quad (20)$$

The proportional control update law is given as

$$K_{Pe}(t) = -e_y(t)e_y^T(t)\bar{\Gamma}_e \quad (21)$$

$$K_{Px}(t) = -e_y(t)x_{ref}^T\bar{\Gamma}_r \quad (22)$$

$$K_{Pu}(t) = -e_y(t)u^T\bar{\Gamma}_u \quad (23)$$

$$\bar{\Gamma}_e, \bar{\Gamma}_r, \bar{\Gamma}_u \geq 0 \quad (24)$$

Here  $\Gamma_e$ ,  $\Gamma_r$  and  $\Gamma_u$  are positive definite scaling matrices (tuning parameters).

Figure 1 below shows a schematic of a model reference adaptive control. It can be noted here that in this paper, the references trajectories are generated only for the positions. Therefore,  $y_{ref}$  outputs reference positions only. In future work, reference control and velocities will be included as well.

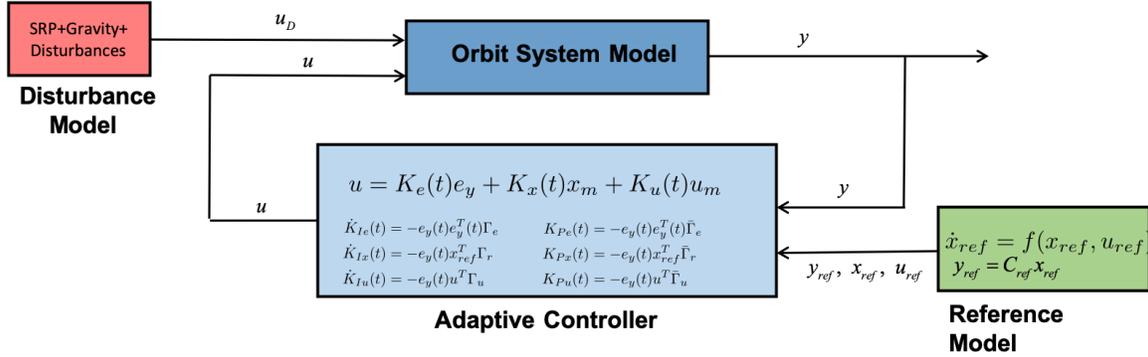


Fig. 1 Adaptive control scheme

## V. Stability Analysis of Non-Linear Model with Direct Adaptive Controller

### A. Dynamical System

The nonlinear dynamics of the model are modified to be given in following state-space representation [1]

$$\dot{x}_p(t) = A_p(x_p)x_p(t) + B_p(x_p)u_p(t) \quad (25)$$

$$y_p(t) = C_p(x_p)x_p(t) \quad (26)$$

It can be noted here that  $A_p$  is nonlinear state dependent matrix. Here  $x \in R^n$ ,  $u \in R^m$ ,  $y \in R^m$ . One of the primary limitations of this type of adaptive controller is that the number states to be controlled must equal to the number of control inputs ( $length(y) = length(u)$ ) which could be an issue for under actuated systems.

Since direct adaptive controllers are model free (independent of dynamics) sufficient conditions must be satisfied by model to guarantee stability. Such conditions are discussed in the following subsections

### B. Definitions

Here we discuss definitions required to prove stability for a direct adaptive control.

### 1. Strictly Passive Systems

A nonlinear system  $(A(x_p), B(x_p), C)$  is called strictly passive (SP) if there exists two positive symmetric matrices (PDS)  $P$  and  $Q$  such that the following two conditions are satisfied

$$\dot{P} + PA + A^T P = -Q \quad (27)$$

$$PB = C^T \quad (28)$$

*Proof:* From [1] it can be shown that equation (27) is the Lyapunov differential equation and the SP system is asymptotically stable. From relation (28) it can be shown that

$$B^T P B = B^T C^T = (CB)^T = CB \quad (29)$$

Therefore, for a SP system  $CB$  is uniformly PDS. However, it must be noted here that most systems are not naturally strictly passive. In other words, they are not naturally stable.

The following formulation is used for non SP systems.

### 2. Almost Strictly Passive System

A system similar to (25) is defined as almost strictly passive (ASP) system if there exists  $P, Q, \tilde{K}_e$ , where  $\tilde{K}_e$  is a constant output feedback gain, unknown and not needed for implementation) such that the closed loop system [5]

$$\dot{P} + P(A - B\tilde{K}_e C) + (A - B\tilde{K}_e C)^T P = -Q \quad (30)$$

$$PB = C^T \quad (31)$$

### 3. Minimum Phase

Apart from passivity of the nonlinear systems, minimum phase property of the system is crucial to prove the stability of the adaptive controls. For a Linear Time In-varying (LTIV) systems stability of zeros is sufficient to prove if the system is minimum phase. However, for a nonlinear system this method is not possible since the transfer function is not defined for those systems.

The system (25) and (4) can be said to be minimum phase if there exists  $M_{n,n-m}$  and  $N_{n-m,n}$  satisfying the following condition

$$CM = 0 \quad (32)$$

$$NB = 0 \quad (33)$$

$$NM = I_{n-m} \quad (34)$$

Here it is assumed that  $B$  and  $C$  are non-trivial, such that  $M$  and  $N$  are well defined. See [1] for proof.

## C. Ideal System Model

When perfect tracking happens i.e.  $y_{ref} = y$ . In other words, when the system model follows the reference trajectory perfectly, the trajectories generated by the system are called ideal trajectories  $(x_p^*(t), u_p^*(t), y_p^*(t))$  and the system model is called ideal model  $(A^*(x_p^*))$ . The model is defined as follows

$$\dot{x}_p^*(t) = A_p^*(x_p^*)x_p^*(t) + B_p^*(x_p^*)u_p^*(t) \quad (35)$$

$$y_p^*(t) = C_p(x_p^*)x_p^*(t) \quad (36)$$

Note: The adaptive control is not necessarily tracking all the system states  $(x_p(t))$ , therefore, the ideal model corresponds to the complete system behaviour when perfect tracking is possible. In other words, during perfect tracking, the system will evolve like an ideal system. It can be noted here that  $y_p^*(t) = y_{ref}$ .

We assume that  $x_p^*(t)$  and  $u_p^*(t)$  are the linear combinations of  $x_{ref}(t)$  and  $u_{ref}(t)$

$$x_p^*(t) = Xx_{ref}(t) + Uu_{ref}(t) \quad (37)$$

$$u_p^*(t) = \tilde{K}_{x_{ref}}x_{ref}(t) + \tilde{K}_{u_{ref}}u_{ref}(t) \quad (38)$$

$$y_p^*(t) = C_p x^*(t) \quad (39)$$

$$= C_{ref}x_{ref}(t) \quad (40)$$

where  $\tilde{K}_{x_{ref}}$  and  $\tilde{K}_{u_{ref}}$  are constant unknown gains for the ideal system.

## D. Error Dynamics and Control Structure

### 1. Error Dynamics

A condition for stability is such that the system states must converge to the ideal states. The error can be defined as follows

$$e_x(t) = x_p^*(t) - x_p(t) \quad (41)$$

Differentiating equation (41) gives

$$\dot{e}_x(t) = \dot{x}_p^*(t) - \dot{x}_p(t) \quad (42)$$

Equation (42) is called the error dynamics equation. The goal adaptive control is to converge the error to zero ( $e_x(t) \rightarrow 0$ ).

## E. Stability Proof

In this section we present the proof for stability for direct adaptive control of structure defined in (13). Substituting ideal system model equation (35) and system model equation (25) in the error dynamics equation (42)

$$\dot{e}_x(t) = A_p^*(x_p^*)x_p^*(t) + B_p^*(x_p^*)u_p^*(t) - A_p(x_p)x_p(t) - B_p(x_p)u_p(t) \quad (43)$$

Adding and subtracting  $A_p(x_p)x_p^*(t)$  in (43)

$$\begin{aligned} \dot{e}_x(t) = & A_p^*(x_p^*)x_p^*(t) + B_p^*(x_p^*)u_p^*(t) - A_p(x_p)x_p(t) - B_p(x_p)u_p(t) \\ & - A_p(x_p)x_p^*(t) + A_p(x_p)x_p^*(t) \end{aligned} \quad (44)$$

Substituting the control law (13) in (44)

$$\begin{aligned} \dot{e}_x(t) = & A_p^*(x_p^*)x_p^*(t) + B_p^*(x_p^*)u_p^*(t) - A_p(x_p)x_p(t) - B_p(x_p)u_p(t) \\ & - A_p(x_p)x_p^*(t) + A_p(x_p)x_p^*(t) \end{aligned} \quad (45)$$

Substituting the ideal system equation (37), ideal system control law (38) Also, adding and subtracting  $B_p\tilde{K}_{e_y}y(t)$ ,  $B_p\tilde{K}_{x_{ref}}x_{ref}(t)$  and  $B_p\tilde{K}_{u_{ref}}u_{ref}(t)$  in above equation (45)

$$\begin{aligned} \dot{e}_x(t) = & A_p(x_p^*(t) - x_p(t)) - A_p(Xx_{ref} + Uu_{ref}) + A_p^*(Xx_{ref} + Uu_{ref}) \\ & - B_pK(t)r(t) + B_p\tilde{K}_{e_y}e_y(t) + B_p\tilde{K}_{x_{ref}} + B_p\tilde{K}_{u_{ref}}u_{ref}(t) \\ & - B_p\tilde{K}_{e_y}e_y(t) - B_p\tilde{K}_{x_{ref}}x_{ref}(t) - B_p\tilde{K}_{u_{ref}}u_m(t) \\ & + B_p^*(\tilde{K}_{x_{ref}}x_{ref} + \tilde{K}_{u_{ref}}u_{ref}) \end{aligned} \quad (46)$$

Combining terms and substituting system error equation (41) in (46)

$$\begin{aligned}
\dot{e}_x(t) = & A_p e_x - B_p \tilde{K}_{e_y} e_y(t) - B_p [K(t) - \tilde{K}] r(t) + [(A_p^* - A_p)X \\
& + (B_p^* - B_p) \tilde{K}_{x_{ref}}] x_{ref}(t) + [(A_p^* - A_p)U \\
& + (B_p^* - B_p) \tilde{K}_{u_{ref}}] u_{ref}(t)
\end{aligned} \tag{47}$$

Assuming that the system parameters vary slowly compared to the control input

$$A_p^* = A_p \tag{48}$$

$$B_p^* = B_p \tag{49}$$

Using assumption (48) and (49) in (47)

$$\dot{e}_x(t) = A_p(x_p) e_x(t) - B_p \tilde{K}(t) e_y(t) - B_p [K(t) - \tilde{K}] r(t) \tag{50}$$

Using the Lyapunov direct method, the Lyapunov function is defined as following

$$V(t) = e_x^T P e_x + tr[K_I - \tilde{K}] \Gamma^{-1} [K_I - \tilde{K}]^T \tag{51}$$

Here,  $e_x, P, K_I$  are function of time and  $\tilde{K}, \Gamma$  are constant.

Differentiating  $V(t)$  and substituting  $\dot{e}_x$  from (50). Alos,  $\dot{K}_I = e_y(t) r(t)^T \Gamma$

$$\begin{aligned}
\dot{V}(t) = & (e_x^T A_p^T - (B_p \tilde{K}_{e_y} e_y)^T - [B_p (K - \tilde{K}) r(t)]^T) P e_x + e_x^T \dot{P} e_x \\
& + e_x^T P (A_p e_x - B_p \tilde{K}_{e_y} e_y(t) - B_p [K(t) - \tilde{K}] r(t)) \\
& + 2tr[(K(t) - e_y(t) r(t)^T - \tilde{K}) \Gamma^{-1} e_y(t) r(t)^T \Gamma]
\end{aligned} \tag{52}$$

Using the ASP property and substituting (30) and (31) in (52) and after some algebra

$$\dot{V}(t) = -e_x^T(t) Q(t) e_x(t) - 2e_y^T(t) e_y(t) r(t)^T \Gamma r(t) \tag{53}$$

From ASP property, the first term of above equation (53) is negative definite and using the La'Salle's principle it can shown that the system is asymptotically stable.

## VI. Simulation

To validate the control algorithm, simulation of a spacecraft around asteroid Ida is considered. For tracking, any smooth reference trajectory can be considered. In this case a closed orbit trajectory  $x_{ref}(t) = (x_r(t), y_r(t), z_r(t))^T$  is given as

$$\begin{aligned}
x_r(t) &= 0.5 R_d \sin(\omega_e t) \\
y_r(t) &= R_d \cos(\omega_e t) \\
z_r(t) &= R_d \sin(\omega_e t)
\end{aligned} \tag{54}$$

Where  $\alpha = 1 \times e^{-8}$ ,  $R_d = 45 \text{ km}$  and  $\omega_e = 1.974\omega$ . Here  $R_d$  is the desired orbit radius and  $\omega$  is rotational velocity of the asteroid Ida. Reference trajectory (54) produces a time dependent smooth trajectory for the adaptive control for tracking.

However, for a general case the spacecraft is initialized closer to the orbit and a full desired trajectory for the maneuver from the initial position and velocity to a final closed loop orbit is given from [10]

$$x_c(t) = x_{in} e^{\alpha t^3} + x_{ref}(t)(1 - e^{\alpha t^3}) \tag{55}$$

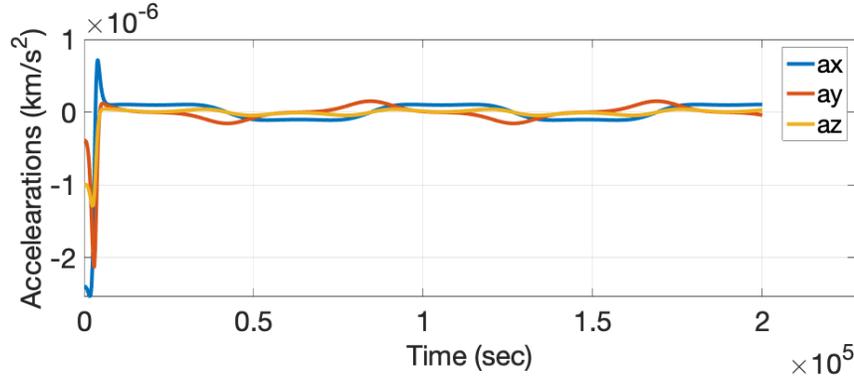
$x_c(t)$  eventually converges to  $x_{ref}(t)$ . Here  $x_{in}$  is the initial position of the spacecraft. The full reference model trajectory is shown in Figure 2.

The initial conditions are given by  $x_{in} = [26 \ 2 \ 5]^T$  (Km) and  $\dot{x}_{in} = [-0.00136 \ 0.000105 \ 0.00114]^T$  (Km/s). The run time is taken to be 20000 seconds. The integral adaptive gains are initialized with zero initial condition. The adaptive gains are given as  $\Gamma_e = \bar{\Gamma}_e = 10^6 I_{3 \times 3}$  and  $\Gamma_r = \bar{\Gamma}_r = 0.001 I_{3 \times 3}$ . Properties of asteroid Ida are taken from [8] and are shown below,

**Table 1 Asteroid Ida Parameters**

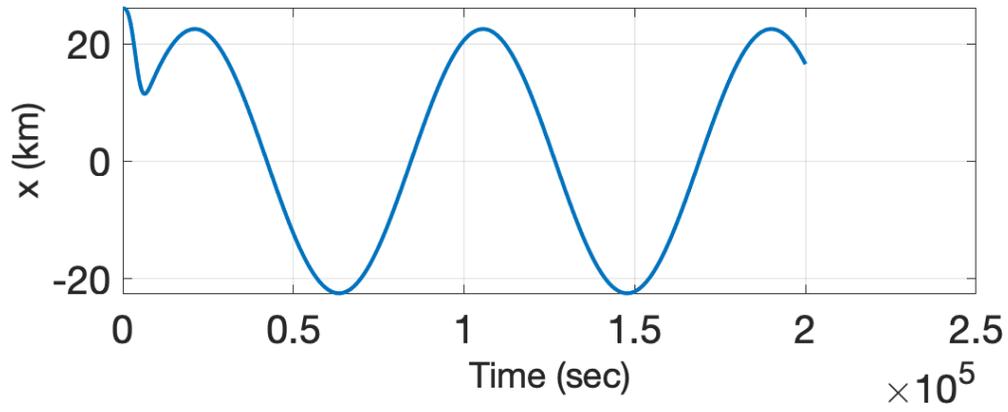
Asteroid Ida Parameters	
Body mass m	$5.1732 \times 10^{16}$ kg
$I_A$	50.85m kg.km <sup>2</sup>
$I_B$	178.5m kg.km <sup>2</sup>
$I_C$	1856m kg.km <sup>2</sup>
$\omega$	$3.77 \times 10^{-4}$ rad/s

Figure 2 below shows the acceleration due to the gravitational perturbations in the body-fixed frame. It can be seen here that the due to the non-spherical nature of the asteroid the gravitational parameters are changing with time.

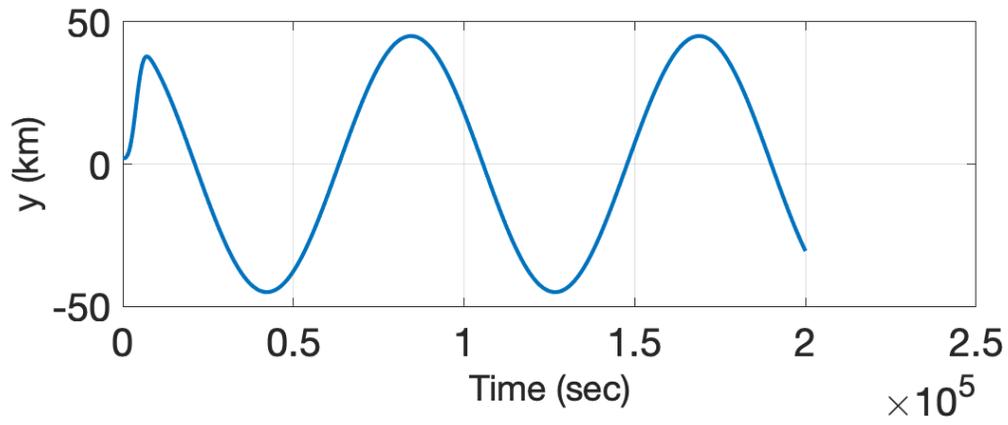


**Fig. 2 Perturbation acceleration due the effect of J2**

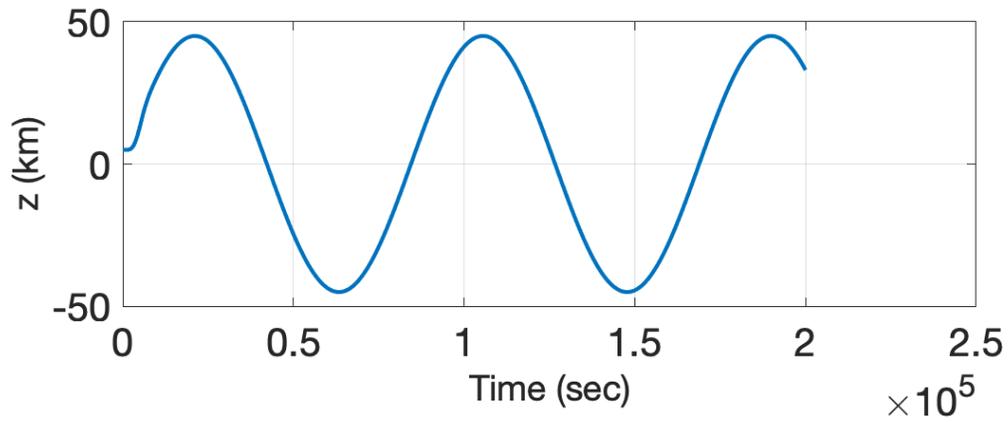
Figure 3, 4, and 5 shows the trajectory of controlled spacecraft in body-fixed frame. It can be seen that after the initial change due to the added term in (55) the positions become time periodic as given by (54).



**Fig. 3** Position of spacecraft in x (km) direction

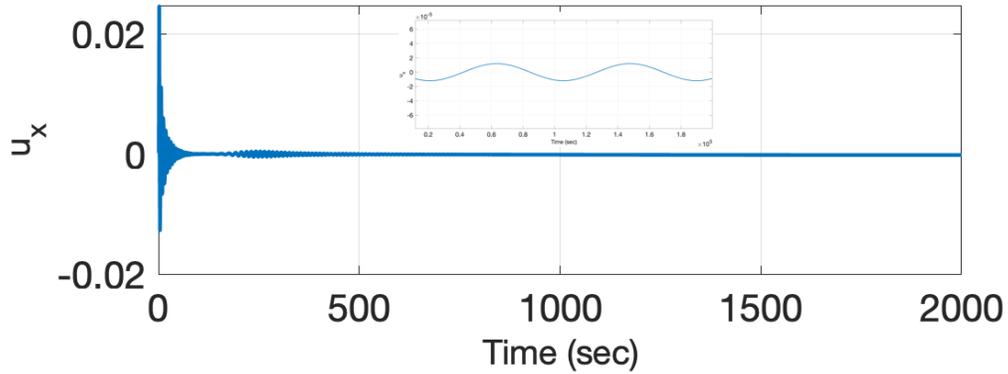


**Fig. 4** Position of spacecraft in y (km) direction

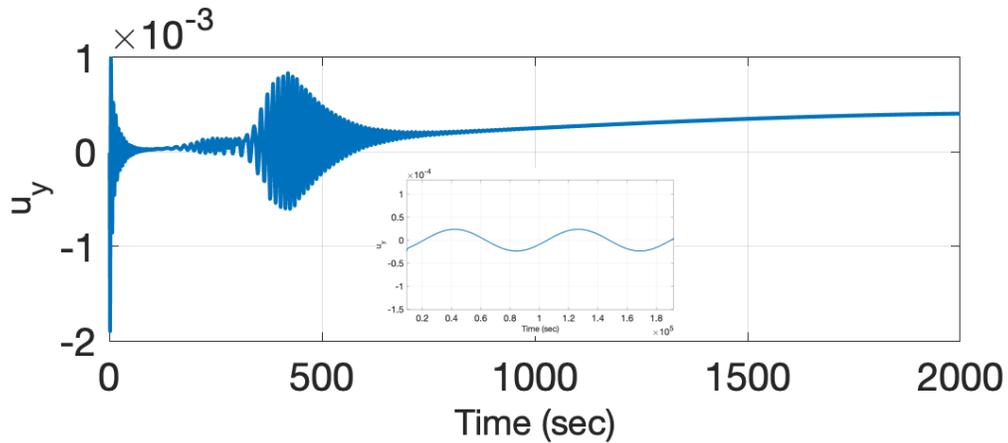


**Fig. 5** Position of spacecraft in z (km) direction

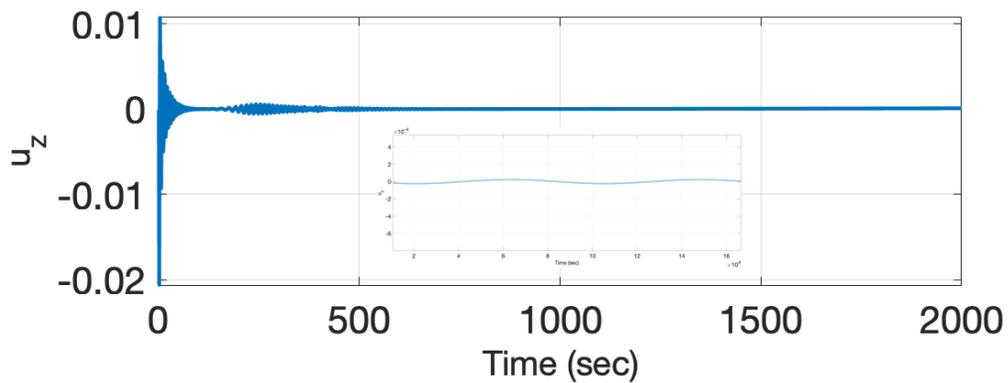
Figures 6, 7, and 8 shows the control effort plots for given reference trajectories. It can be that the control effort peaks at initially and then quickly settles. This peak can be explained by the fact that reference trajectories are not tracking any control effort which not only results in these peaks, but also fast oscillations. Both of these issues are planned to be resolved by tracking optimal reference trajectories. It must also be noted here that due to the periodic nature of the gravitational accelerations non-zero control effort required to keep track the orbit. The zoomed in part is shown in the control efforts plots.



**Fig. 6 Control effort in x ( $Km/s^2$ )**

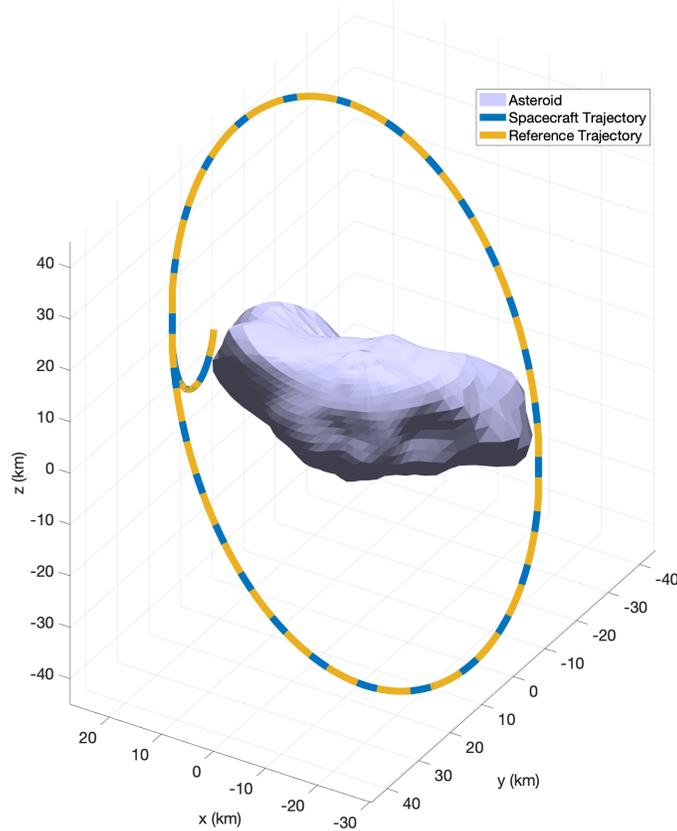


**Fig. 7 Control effort in y ( $km/s^2$ )**



**Fig. 8 Control effort in z ( $km/s^2$ )**

Finally, Figure 9 shows the overlap between reference and controlled trajectory. It can be seen that the adaptive controller is able to successfully track the given reference trajectory. Other time dependent trajectories can be used to track as well. It must noted that  $\alpha$  from (55) plays an important role in numerical integration scheme in MATLAB.



**Fig. 9** Spacecraft trajectory with control

## VII. Conclusion

An adaptive controller based on the simple adaptive control (SAC) algorithm is being developed. Results show that the controller is able to account for the unknown gravitational perturbation and SRP. A non-zero control effort is required to keep the spacecraft in orbit due to time varying gravitational perturbations. As discussed above, unrealistic control switching is required to initially track the trajectories(over 20 Hz). This issue is planned to be resolved using optimal control trajectories.

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