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SPACECRAFT BODY-FRAME HOVERING OVER AN ASTEROID USING A DIRECT ADAPTIVE CONTROL STRATEGY

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In this paper, we have implemented a direct adaptive control strategy for the spacecraft hovering problem in the vicinity of an asteroid. The asteroid and spacecraft parameters, including gravitational parameters, solar radiation pressure, inertias and higher order harmonics, are assumed to be unknown. A fully nonlinear dynamical model with McCullagh's gravitational approximation and solar radiation pressure is implemented. Hovering trajectories are presented in an asteroid fixed body frame. Simulation results show successful trajectory tracking using the direct adaptive control strategy.

INTRODUCTION

Hovering maneuvers are critical to spacecrafts visiting asteroids. The most recent missions to asteroids Bennu and Ryugu by NASA and JAXA performed several hovering maneuvers to study the asteroid and eventually to land and collect samples to bring back to Earth. Usually, there are two types of hovering scenarios: an asteroid body fixed hovering scenario, where the spacecraft position is fixed with respect to the rotating asteroid frame, and an asteroid fixed inertial frame, where the spacecraft position is fixed with respect to the inertially fixed frame. In this paper, we have considered asteroid body fixed hovering scenario. However, this work can be applied to inertial frame hovering as well.

Maneuvering in the vicinity of asteroids presents several challenges. A vast majority of details regarding the asteroids are usually unknown before the mission.[1] Proximity operations around small celestial bodies like asteroids have their own challenges such as irregular gravitational field of the body, effects of solar radiation pressure and communication delays, to name a few. Due to the small size of asteroids, it is impossible to study their gravitational fields in detail from earth based technologies. Therefore, a significant amount of time is spent to study the asteroid characteristics before performing maneuvers in the vicinity of the asteroid. For example, Hayabusa 1,2 and OSIRIS-Rex spent over a month to study and model the characteristics of the asteroids. Another significant cause of disturbance is the solar radiation pressure (SRP) force. It has been shown that in some cases SRP plays a stronger role as compared to the gravitational field when in close proximity to asteroids.[2]

Hence, there is a need to develop autonomous technologies that can counter the effects of unknown models and external disturbances to achieve mission objectives. In the realm of dynamics

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and control, a direct adaptive control strategy could help to resolve some these aforementioned issues. Unlike other model based controllers, such as robust controllers and optimal controllers, a direct adaptive controller does not require estimation of system parameters to achieve tracking, thus reducing the dependency on ground based systems and increasing the robustness and autonomy of the mission. This form of controller could also significantly reduce the mission timeline and, in the future, could help to deploy faster missions to small bodies like asteroids and comets.

Due to recent developments and interests, several researchers have contributed to the area of spacecraft dynamics and controls in the vicinity of small bodies. Stackhouse[3] developed an indirect adaptive control strategy using the Udwadia-Kalaba formulation for hovering in the vicinity of an asteroid. Broschart and Scheeres[4] presented an active control hovering strategy to cancel disturbing acceleration. Furfaro[5] presented a novel nonlinear guidance algorithm using concepts of sliding mode control theory. Lee[6] designed a global asymptotic tracking control for body fixed hovering over an asteroid in the geometric mechanics framework, and a continuous-time feedback control using exponential coordinates is also shown. Lee and Singh[7] developed an immersion and invariance based adaptive control for asteroid orbit and hovering. In this work, a noncertainity-equivalence adaptive control scheme is designed for spacecraft hovering, where the mass and moments of inertia of the asteroid are assumed to be unknown. Direct adaptive control methodology for tracking other reference trajectories around asteroids can be found in Tiwari.[8]

In this paper, a model free direct adaptive controller is implemented to track hovering trajectories in an asteroid fixed body frame and an orbit fixed inertial frame. Asteroid and spacecraft characteristics including gravitational, SRP and mass are assumed to unknown. It is assumed that the asteroid is rotating with a constant angular velocity about a fixed axis. The system model that is implemented in this work is a fully nonlinear dynamical model with an irregular gravitational field and solar radiation pressure (SRP). Simulations result is shown for asteroid Ida. The results show precise orbit-hover tracking in the vicinity of the asteroid.

The paper is divided into the following sections. First, we discuss the formulation of system dynamics, where we present the spacecraft equations of motions with formulation of gravity and solar radiation pressure forces. Then we present the state space form of the control problem followed by the adaptive control structure. Lastly, we present simulation results for the hovering scenario.

SYSTEM DYNAMICS

In this section, we present the dynamical model of the spacecraft with respect to the asteroid fixed body frame. The forces acting on the spacecraft due to gravity and solar radiation pressure are also discussed in detail.

Spacecraft Dynamics

The dynamics of the spacecraft for the hovering scenarios is considered in the asteroid fixed body frame. The body frame is assumed to be aligned with the principal axes of the asteroid and the translational equation of motion is given as:

$$\ddot{\mathbf{r}} + 2\boldsymbol{\Omega} \times \dot{\mathbf{r}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) = \frac{\partial U}{\partial \mathbf{r}} + \mathbf{a}_{srp} + \mathbf{u} \tag{1}$$

where $\mathbf{r} \in R^3$ represents the position vector of the spacecraft with respect the body fixed frame of the asteroid, $\mathbf{\Omega} \in R^3$ is the angular velocity vector of the asteroid in the asteroid fixed body frame. $\mathbf{u} \in R^3$ is the control acceleration vector, $\mathbf{a}_{SRP} \in R^3$ is the acceleration due to solar radiation pressure and $\frac{\partial U}{\partial \mathbf{r}} \in R^3$ is acceleration due to the gravity of the asteroid.

Forces

In this section we present the formulation of forces due to gravity and solar radiation pressure.

Modeling Gravity using MacCullagh's Approximation: MacCullagh's gravity model is a second order approximation of the gravity potential field. This approximation retains only 3 terms of the infinite series of the gravitational potential field. The gravity potential model is written with respect to the moment of inertias of the body. More detail about this model can be found in Schaub.[9]

The final form of the gravitational potential field of an arbitrary body with a body-fixed frame at the center of mass of the body is given as:

$$V(r) = -\frac{Gm}{|\mathbf{r}|} - \frac{G}{2|\mathbf{r}|^3}(I_A + I_B + I_C - 3I_r) \tag{2}$$

$$\tag{3}$$

where

$$I_r = \frac{1}{|r|^2}(I_A x^2 + I_B y^2 + I_C z^2) \tag{4}$$

Here I_r is the polar moment of the body about the parallel to position vector $\mathbf{r} \in R^3$, I_A, I_B and I_C are the moment of inertias of the asteroid with respect to the asteroid body fixed frame, G is the universal gravitational constant and $|\mathbf{r}|$ is the the norm of the position vector of the point subjected to the gravitational potential field. It can be noted here that, for a spherical body $I_A = I_B = I_C = I_r$,which equals the point mass model of the gravity field.[10]

By taking the negative of the gradient of the gravitational potential field $V(\mathbf{r})$, the resulting acceleration on the spacecraft in vector form is given as

$$\frac{\partial U}{\partial \mathbf{r}} = -\nabla V(\mathbf{r}) \tag{5}$$

$$\frac{\partial U}{\partial \mathbf{r}} = -\frac{\mu}{|\mathbf{r}|^3}\mathbf{r} - \frac{3G}{2|\mathbf{r}|^5}(I_A + I_B + I_C - 5I_r)\mathbf{r} - \frac{3G}{|\mathbf{r}|^5}(I_A x\hat{x} + I_B\hat{y} + Iz\hat{z}) \tag{6}$$

Accelerations in x, y and z components with respect to the asteroid fixed body frame can be written as follows

$$a_x = -\frac{\mu}{|\mathbf{r}|^3}x - \frac{3}{2}G(I_A + I_B + I_C - 5I_r) + 3GI_A \tag{7}$$

$$a_y = -\frac{\mu}{|\mathbf{r}|^3}y - \frac{3}{2}G(I_A + I_B + I_C - 5I_r) + 3GI_B \tag{8}$$

$$a_z = -\frac{\mu}{|\mathbf{r}|^3}z - \frac{3}{2}G(I_A + I_B + I_C - 5I_r) + 3GI_C \tag{9}$$

Solar Radiation Pressure (SRP) Model: Solar radiation pressure force has been shown to significantly affect the orbital dynamics of the spacecraft near small bodies. Several types of SRP models exist such as a cuboid model and a flat plate model. It was found that a flat plate model is sufficient to model SRP on a spacecraft with large solar panels since the solar panels have a large span across the spacecraft. Therefore, a flat plate SRP model is adopted in this paper. To model the SRP, two frames of references are introduced which are centered at the center of mass of the spacecraft. From Figure 1, x^{SC}, y^{SC} and z^{SC} are the axes fixed at the center of spacecraft and aligned with principal axes of moment of inertia, and x^{SP}, y^{SP}, and z^{SP} are the axes at the center of mass of the spacecraft. Here z^{SP} is the sun pointing vector and x^{SP} is in the plane formed by the x and y axes of the asteroid frame. More details can be found in Kikuchi.[11]

Without loss of generality, the plate is assumed to be perpendicular to the z^{SC} axis. For getting maximum power output from the solar arrays, the goal would be to align the z^{SC} axis with the sun pointing vector z^{SP}. Therefore, ideally $\hat{\mathbf{z}}^{SC} \cdot \hat{\mathbf{z}}^{SP} = 1$. Here, $\hat{\mathbf{z}}^{SC}$ and $\hat{\mathbf{z}}^{SP}$ are unit vectors corresponding to the z^{SC} and z^{SP} axes. It can be noted that aligning the axes this way results in maximum force due to SRP, thus increasing external disturbances on the spacecraft.

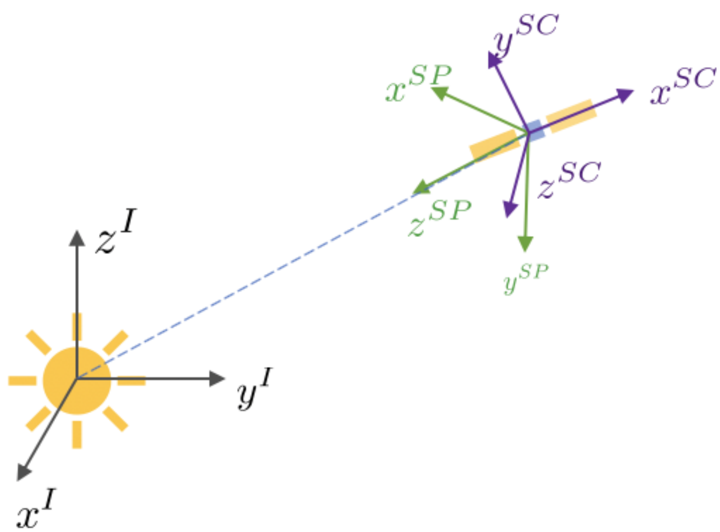

Figure 1. Spacecraft body frame (SC)

For a flat plate model, accelerations due to SRP are given by

$$\mathbf{a}_{SRP} = -\frac{PA}{m}(\hat{\mathbf{z}}^{SC}.\hat{\mathbf{z}}^{SP})[2((\hat{\mathbf{z}}^{SC}.\hat{\mathbf{z}}^{SP})C_s + B_f C_d)\hat{\mathbf{n}} + (C_d + C_a)\hat{\mathbf{z}}^{SP}] \tag{10}$$

where, $B_f = \frac{2}{3}$ is the Lambertian coefficient; A is the surface area of the flat plate; and $P = \frac{P_0}{d^2}$ is the SRP acting on the surface of the spacecraft. Also, $P_0 = 1 \times 10^{17} \frac{kg.m}{s^2}$ and C_s, C_d, C_a corresponds to optical constants of the spacecraft. A much simpler model, as shown below, can be considered as well:

$$\mathbf{a}_{SRP} = -a_0 cos^2(\theta)\hat{\mathbf{n}} \tag{11}$$

where $cos(\theta)$ is the dot product between $\hat{z}^{SC}$ and $\hat{z}^{SP}$, which can be taken as 1 to maximize the disturbance acceleration caused by SRP. To model the attitude dynamics of the spacecraft, Euler equations of rotations are implemented with SRP as the only external force acting on the spacecraft. For modelling attitude dynamics, please refer to Schaub.[9]

State Space Form

For the purposes of this research and without loss of generality, the angular velocity vector of the asteroid is aligned with the z axis of the asteroid. For completeness, the equations in the Cartesian frame are expressed in Equation (12).

$$
\begin{aligned}
\ddot{x} - 2\omega\dot{y} - \omega^2 x &= a_x + a_{SRP_x} + u_x \\
\ddot{y} + 2\omega\dot{x} - \omega^2 y &= a_y + a_{SRP_x} + u_y \\
\ddot{z} &= a_z + a_{SRP_x} + u_z
\end{aligned} \tag{12}
$$

where ω is the magnitude of the angular velocity of the asteroid about the z axis.

The state-space formulation for the dynamical system is given as

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{F}(\mathbf{x}) + \mathbf{B}\mathbf{u} \\
\mathbf{y} &= \mathbf{C}\mathbf{x}
\end{aligned} \tag{13}
$$

where $\mathbf{F}(\mathbf{x})$, $\mathbf{B}$ and $\mathbf{C}$ are given as

$$
\mathbf{F}(\mathbf{x}) = \begin{bmatrix} 2\omega\dot{y} + \omega^2 x + a_x + a_{SRP_x} \\ -2\omega\dot{x} + \omega^2 y + a_y + a_{SRP_x} \\ a_z + a_{SRP_x} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{0}_{3\times3} \\ \mathbf{I}_{3\times3} \end{bmatrix}, \mathbf{C} = [\mathbf{I}_{3\times3} \;\; \mathbf{I}_{3\times3}] \tag{14}
$$

The system given in Equation (13) can be expressed in the following square state-space form as follows

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}\mathbf{u}, \qquad && \mathbf{x} \in R^6 \\
\mathbf{y} &= \mathbf{C}\mathbf{x} && \mathbf{y} \in R^3
\end{aligned} \tag{15}
$$

This form in Equation (15) is exploited to study the stability of the adaptive controller. See the work from Barkana[12] for more details. It can be noted here that $\mathbf{C}$ from Equation (14) outputs the states as the summation of position and velocity, also known as blended output. This has shown to significantly improve the control efforts by adding damping effects on the control thrust.

CONTROL STRUCTURE

A direct adaptive controller, also known as a simple adaptive controller (SAC), is implemented in this paper. The form of the control is given in Equation (16). Unlike many forms of adaptive controllers such as indirect adaptive controllers, this form of controller is a model free controller that assumes no prior knowledge about the underlying dynamical system. Therefore, a mathematical model of the system dynamics is not required.

SAC is an output feedback controller and only require the states that are tracked to be observed. The adaptive controller is given as

$$
\mathbf{u} = \mathbf{K}_e(t)\mathbf{e}_y + \mathbf{K}_{\mathbf{x}}(t)\mathbf{x}_m + \mathbf{K}_{\mathbf{u}}(t)\mathbf{u}_m \tag{16}
$$

Here $\mathbf{e}_y$ is the output tracking error defined as

$$\mathbf{e}_y = \mathbf{y}_m - \mathbf{y} \quad (17)$$
$$\mathbf{e}_y = \mathbf{C}\mathbf{x}_m - \mathbf{C}\mathbf{x} \quad (18)$$

where $\mathbf{y}_m$ and $\mathbf{y}$ are the output vectors of the reference and actual models respectively.

$\mathbf{K}_e(t) \in R^{m\times m}$ is the time-varying control gain matrix, $\mathbf{K}_x(t) \in R^{m\times n}$ and $\mathbf{K}_u(t) \in R^{m\times m}$ are time-varying feedforward control gains, and $\mathbf{x}_m$ and $\mathbf{u}_m$ are the states and control vectors of the reference model.
The adaptive gains $\mathbf{K}_e$, $\mathbf{K}_u$ and $\mathbf{K}_u$ from Equation (16) are given as the summation of integral adaptive gains $\mathbf{K}_I = [\mathbf{K}_{Ie}\ \mathbf{K}_{Ix}\ \mathbf{K}_{Iu}]$ and proportional adaptive gains $\mathbf{K}_P = [\mathbf{K}_{Pe}\ \mathbf{K}_x\ \mathbf{K}_{Pu}]$ as follows:

$$\begin{aligned} \mathbf{K}_e(t) &= \mathbf{K}_{Ie}(t) + \mathbf{K}_{Pe}(t) \\ \mathbf{K}_x(t) &= \mathbf{K}_{Ix}(t) + \mathbf{K}_{Px}(t) \\ \mathbf{K}_u(t) &= \mathbf{K}_{Iu}(t) + \mathbf{K}_{Pu}(t) \end{aligned} \quad (19)$$

Kaufman,[13] to guarantee stability and achieve desired tracking performance, has shown that only the integral gains $\mathbf{K}_I$ are necessary. However, proportional adaptive gains $\mathbf{K}_P$ have been shown to improve the rate of convergence of the SAC. Hence, they have been implemented in this paper.

The integral control update law is given as

$$\begin{aligned} \dot{\mathbf{K}}_{Ie}(t) &= -\mathbf{e}_y(t)\mathbf{e}_y^T(t)\mathbf{\Gamma}_e \\ \dot{\mathbf{K}}_{Ix}(t) &= -\mathbf{e}_y(t)\mathbf{x}_{ref}^T\mathbf{\Gamma}_r \\ \dot{\mathbf{K}}_{Iu}(t) &= -\mathbf{e}_y(t)\mathbf{u}^T\mathbf{\Gamma}_u \end{aligned} \quad (20)$$

Here $\mathbf{\Gamma}_e$, $\mathbf{\Gamma}_r$ and $\mathbf{\Gamma}_u$ are positive definite weighting matrices (tuning parameters) for the integral adaptive control law. It can noted that these parameters must be manually tuned. Some guidance on how to tune these parameters is provided in Kaufman.[13]

The proportional control update law is given as

$$\begin{aligned} \mathbf{K}_{Pe}(t) &= -\mathbf{e}_y(t)\mathbf{e}_y^T(t)\bar{\mathbf{\Gamma}}_e \\ \mathbf{K}_{Px}(t) &= -\mathbf{e}_y(t)\mathbf{x}_{ref}^T\bar{\mathbf{\Gamma}}_r \\ \mathbf{K}_{Pu}(t) &= -\mathbf{e}_y(t)\mathbf{u}^T\bar{\mathbf{\Gamma}}_u \end{aligned} \quad (21)$$

Here, $\bar{\mathbf{\Gamma}}_e$, $\bar{\mathbf{\Gamma}}_r$ and $\bar{\mathbf{\Gamma}}_u$ are positive semi-definite matrices used to tune the proportional adaptive gain laws.

To reduce the complexity of the adaptive control system and to avoid generating the reference control inputs $\mathbf{u}_r$, a modified version of the control law is implemented where the feed-forward reference control $\mathbf{u}_r$ is set to zero. It can be noted from Kaufman[13] that the feed-forward reference control is not necessary to guarantee stability.

The modified adaptive control is then given as

$$\mathbf{u} = \mathbf{K}_e(t)\mathbf{e}_y + \mathbf{K}_{\mathbf{x}}(t)\mathbf{x}_m \quad (22)$$

Figure 2 shows a schematic of a model reference adaptive controller. It can be noted here that in this paper, the references trajectories are generated for the combination of position and velocity. However, in general, the adaptive controller does not require full state feedback, which is another advantage of this controller.

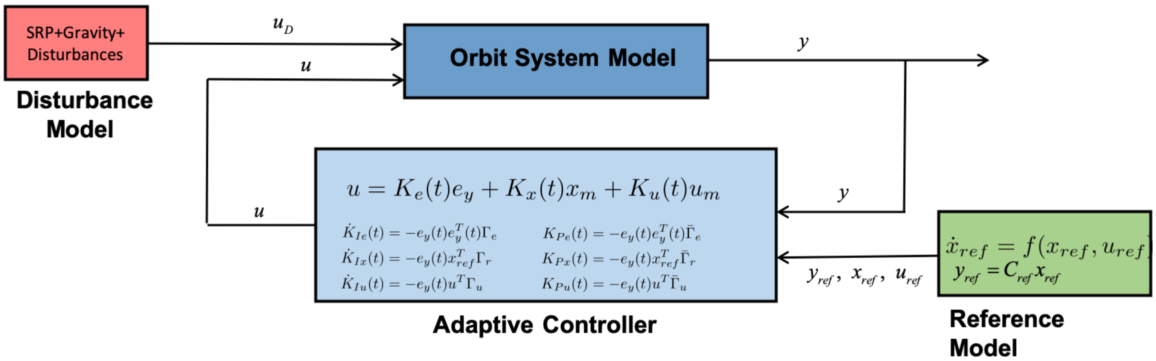

Figure 2. Adaptive Control System Schematic

Stability Conditions

Here we present the criteria that are necessary to guarantee the stability of the nonlinear dynamical system with the direct adaptive control. Refer to Barkana[12] for a complete stability proof.

Condition 1: For linear systems, stability of zero-dynamics can be shown by writing the transfer functions. However, for nonlinear systems, transfer functions are not defined. Therefore, the concept of zero-dynamics is extended by defining matrices $\mathbf{M}(\mathbf{x}, t) \in R^{m,n-m}$ and $\mathbf{N}(\mathbf{x}, \mathbf{t}) \in R^{n-m,n}$ such that:

$$\begin{aligned} \mathbf{CM} &= \mathbf{0} \\ \mathbf{NB} &= \mathbf{0} \\ \mathbf{NM} &= \mathbf{I}_{n-m} \end{aligned} \tag{23}$$

Here, m and n are the length of vectors state vector $\mathbf{x}$ and output vector $\mathbf{y}$, respectively. Therefore, if there exist matrices $\mathbf{M}(\mathbf{x}, t)$ and $\mathbf{N}(\mathbf{x}, \mathbf{t})$ such that the relationships in Equation (23) are satisfied, then the nonlinear system defined in Equation (15) is defined to be minimum-phase (see Barkana[12]).

For the dynamical system given in Equation (13), the relationships from Equation (23) are satisfied by defining $\mathbf{M}(\mathbf{x}, t)$ and $\mathbf{N}(\mathbf{x}, \mathbf{t})$ as:

$$\mathbf{M}(\mathbf{x}, t) = \begin{bmatrix} \mathbf{I}_{3\times3} \\ -\mathbf{I}_{3\times3} \end{bmatrix}, \quad \mathbf{N}(\mathbf{x}, t) = \begin{bmatrix} \mathbf{I}_{3\times3} & -\mathbf{0}_{3\times3} \end{bmatrix} \tag{24}$$

It can be seen that Equation (24) satisfies the relationships given in Equation (23). Note the choice of matrices $\mathbf{M}(\mathbf{x}, t)$ and $\mathbf{N}(\mathbf{x}, \mathbf{t})$ is non-unique and any non-trivial and well defined matrices can be chosen as long as the relationship given in Equation (23) are satisfied. To guarantee stability, the system must be minimum-phase and the conditions shown above must be satisfied.

Condition 2: A system as shown in Equation (15), which is minimum-phase and satisfies the condition that the product $\mathbf{CB}$ is positive-definite and symmetric (PDS), is guaranteed to be asymptotically stable (see Barkana[12] for the proof).

For the dynamical system given in Equation (13), it can shown that the product $\mathbf{CB}$ is PDS as follows

$$\mathbf{CB} = \begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{I}_{3\times3}\end{bmatrix}\begin{bmatrix}\mathbf{0}_{3\times3}\\ \mathbf{I}_{3\times3}\end{bmatrix} > \mathbf{0}_{3\times3} \tag{25}$$

Therefore, the dynamical system with the direct adaptive control methodology as given in Equation (15) satisfies the conditions required to guarantee stability. When these conditions are met, the system is also known as an almost strictly passive (ASP) system.

SIMULATION AND RESULTS

In this section, we will the discuss the formulation of the reference model implemented and present simulation results using the adaptive control algorithm.

Reference Model

The reference trajectory for hovering is designed in the asteroid fixed body frame. The spacecraft is required to track a time-varying trajectory from the initial position to the final hovering position. The reference trajectory is given as

$$\mathbf{x}_m = \mathbf{x}_{in}e^{\alpha t^3} + \mathbf{x}_f(1 - e^{\alpha t^3}) \tag{26}$$

$$\mathbf{y}_m = \mathbf{C}\mathbf{x}_m \tag{27}$$

where $\mathbf{y}_m \in R^3$ is the time-varying output equation for the spacecraft to track. The tuning parameter $\alpha = -1 \times e^{-8}$ corresponds to the rate of change of position which can be tuned depending on time constraints. $\mathbf{x}_{in} = [\mathbf{x}_{inp} \;\; \mathbf{x}_{inv}]$ is the initial position and velocity of the spacecraft and the $\mathbf{x}_f = [\mathbf{x}_{fp} \;\; \mathbf{x}_{fv}]$ is the final spacecraft hovering position and velocity in the asteroid fixed body frame. The reference trajectory from Equation (26) generates a time-varying smooth trajectory for the adaptive controller for tracking. It can be noted here that the direct adaptive controller of the form of Equation (16) requires a time-varying trajectory to track between initial and final states.

Simulation Properties

For the purposes of this research, asteroid Ida is chosen for the simulations presented in this paper. A rest-to-rest maneuver is simulated. The properties of asteroid Ida can be found in Table 1.

Table 1. Asteroid Ida Parameters

Body mass (M)	5.1732×10^{16} kg
I_A	50.85M kg.km^2
I_B	178.5M kg.km^2
I_C	185.6M kg.km^2
ω	3.77×10^{-4} rad/s

The initial and the final positions for the reference model are chosen as

$$\begin{aligned}\mathbf{x}_{inp} = [4\ 12\ 6\ ]^T \text{ km},\ \ \mathbf{x}_{inv} = [0\ 0\ 0]^T \times 10^{-3}\text{km/s}\\ \mathbf{x}_{fp} = [5\ 25\ 6\ ]^T \text{ km},\ \ \mathbf{x}_{fp} = [0\ 0\ 0\ ]^T \text{ km/s}\end{aligned} \tag{28}$$

The initial state for the system model is chosen as

$$\mathbf{x}_{0p} = [4\ 12\ 6\ ]^T \text{ km},\ \ \mathbf{x}_{0v} = [0\ 0\ 0]^T \times 10^{-3}\text{km/s} \tag{29}$$

where $\mathbf{x}_{0p}$ and $\mathbf{x}_{0v}$ are the initial position and velocity of the the system model. It can noted that the reference model initial condition matches with the initial condition of the system. The integral adaptive gains are initialized with zero initial condition. The adaptive tuning parameters are given as $\boldsymbol{\Gamma}_e = \bar{\boldsymbol{\Gamma}}_e = 10^6 I_{3\times3}$ and $\boldsymbol{\Gamma}_r = \bar{\boldsymbol{\Gamma}}_r = 0.001 I_{3\times3}$.

Simulation Results

The Spacecraft trajectory as seen from the inertial frame is shown in Figure 3. It can be seen from Figure 3 that the spacecraft successfully translated from the initial position to the desired final hovering position. Figure 4 shows the spacecraft position in the asteroid fixed body frame. The

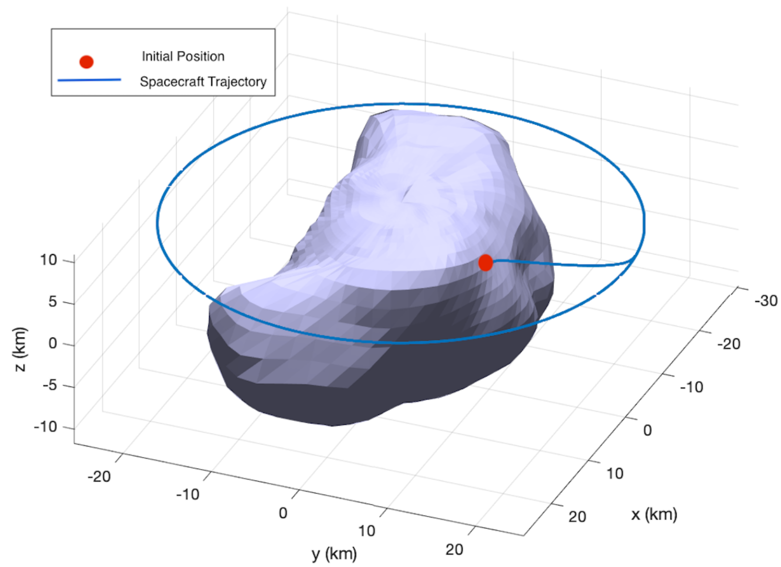

Figure 3. Spacecraft position in inertial frame

adaptive controller is able to successfully hover at the desired position with respect to the asteroid.

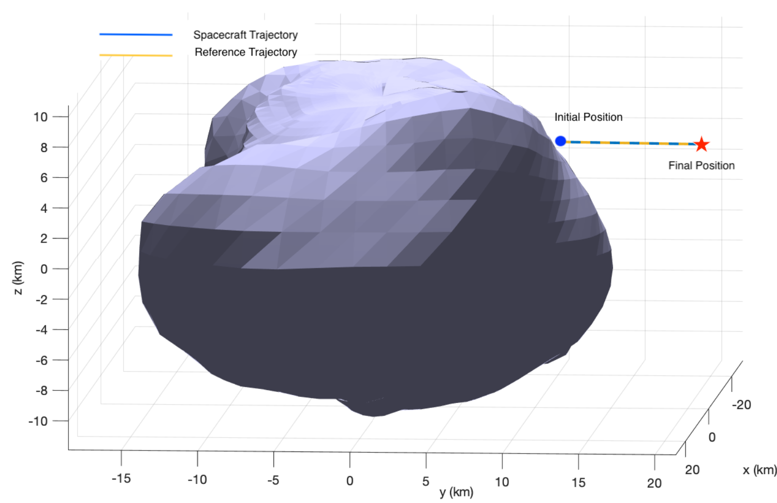

Figure 4. Spacecraft position in asteroid body frame

In the body fixed hovering scenario, the accelerations due to gravity from the asteroid are time invariant. This is due to the fact that the position of the spacecraft is fixed with respect to the asteroid. Figure 5 shows the total acceleration acting on the spacecraft due to the asteroid gravity field. It can be noted that non-zero control effort is required for station keeping at the desired location. Figure 6 shows the time-varying trajectory of the spacecraft in the asteroid fixed body frame.

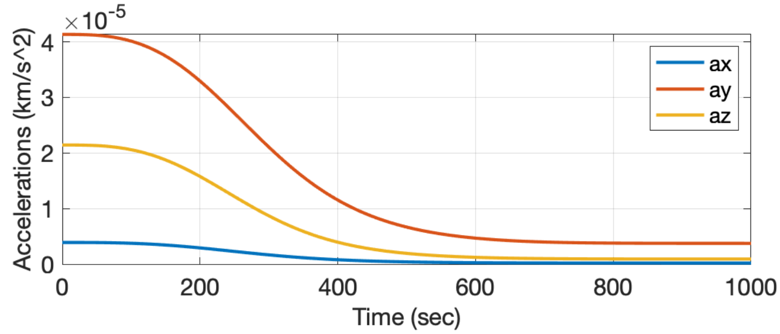

Figure 5. Accelerations due to the McCullagh gravity model (in asteroid body frame)

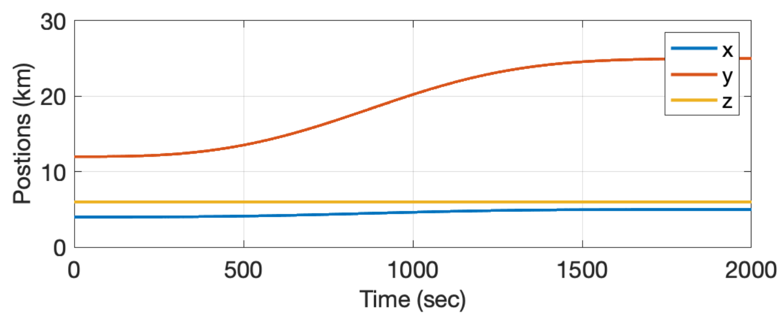

Figure 6. Positions with respect to time (in asteroid body frame)

The control effort from the direct adaptive controller can be seen in Figure 7. At the final hovering position, the control effort converges to the summation of acceleration due to gravity and solar radiation pressure. The control effort resulting from this problem can be further reduced depending on the the trajectory and tuning α in Equation (26). It can be noted that, since the y coordinate of the spacecraft has the largest change in position, the control effort $\mathbf{u}_y$ is also the highest. Further command shaping can reduce this control effort. Also, the control effort is within the acceptable range that can be produced with currently available chemical thrusters. The final hovering is approximately reached in 1696 seconds ($\sim$ 30 minutes). The maximum magnitude of the control effort is $\mathbf{u}_{max} = 3.4320 \times 10^{-5}$ km/s^2. To capture the effect of solar radiation pressure force during station keeping at the final hovering position, the spacecraft is left to freely rotate under the effect of SRP. Figure 8 shows the acceleration due to the presence of SRP at the final hovering position. It can be seen that a non-zero and time-varying control effort must be required to keep the spacecraft at the hovering position. It can be noted from the comparing Figure 5 and Figure 8 that the order of magnitude of the gravity accelerations and SRP is similar.

The control efforts resulting from SRP and gravity can be captured after reaching the final hovering position. Figure 9 shows the long term adaptive control effort for the complete trajectory. It can be noted that a non-zero and time-varying control effort is required for station keeping at the

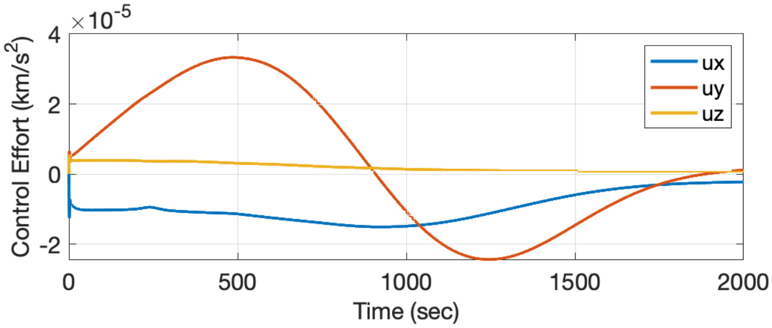

Figure 7. Adaptive control effort (in asteroid body frame)

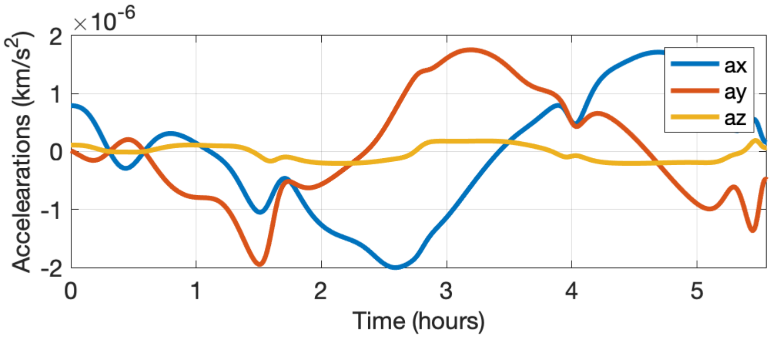

Figure 8. Accelerations due to SRP (in asteroid body frame)

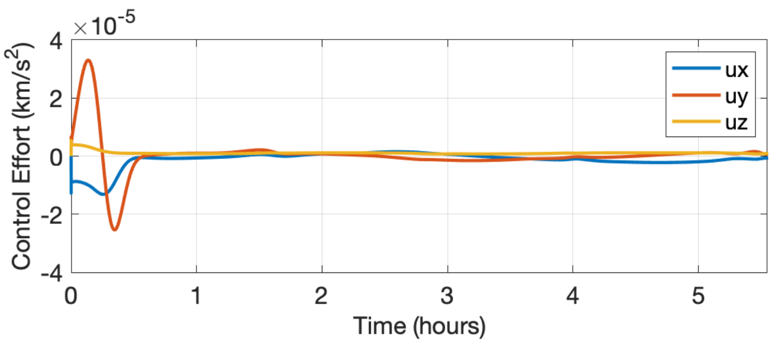

Figure 9. Long term adaptive control effort (with SRP) (in asteroid body frame)

final hovering position. In the absence of SRP, the control efforts reach a non-zero but constant control effort. This is due to the fact that the only external force acting on the spacecraft at the final hovering state ($\mathbf{x}_f$) is the gravity due to the asteroid.

The long term control efforts without SRP can be seen in Figure 10. It can seen that the control efforts converge to a constant value due to the absence of SRP force on the spacecraft. The output tracking error ($\mathbf{e}_y$) with respect to time can be seen in Figure 11 and Figure 12. Figure 11 shows the output error during initial phase of the tracking. Initially there is an overshoot in error primarily due to the initialization of adaptive control gains as zero. Once the gains adapt, the tracking error is significantly reduced as shown in Figure 12. Magnified output error can be seen in Figure 13.

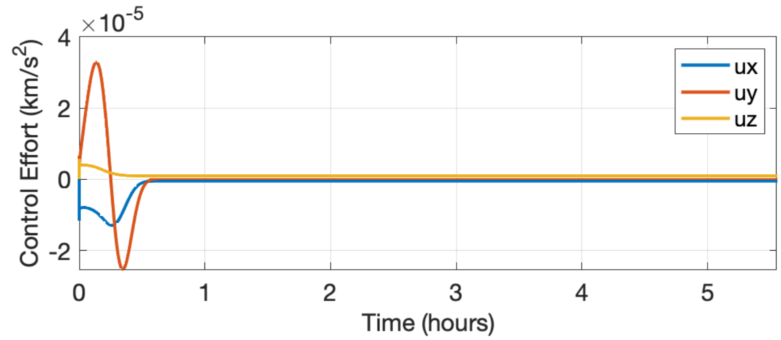

Figure 10. Long term adaptive control effort (without SRP) (in asteroid body frame)

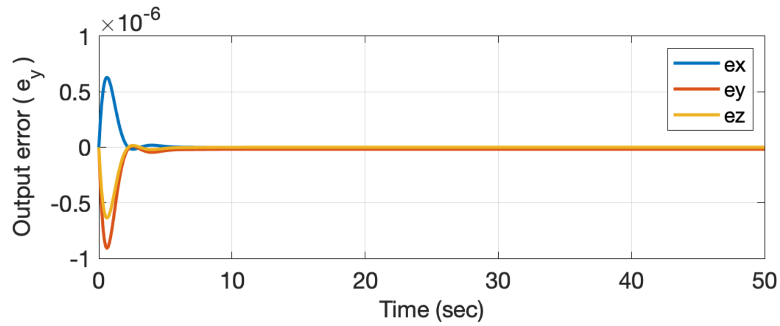

Figure 11. Initial output error (in asteroid body frame)

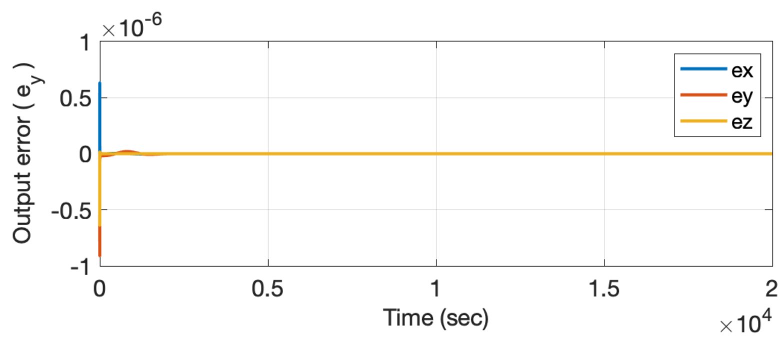

Figure 12. Output Error (in asteroid body frame)

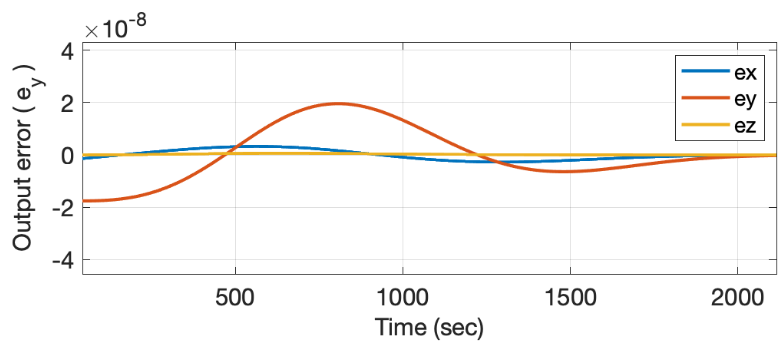

Figure 13. Magnified Output Error (in asteroid body frame)

CONCLUSION

In this paper, a direct adaptive controller was developed to track trajectories in the asteroid fixed body frame in the presence of unknown disturbances. An important advantage of this controller is that it does not require estimation of the system parameters. Hence, it can be proven to be effective in environments where it is difficult to estimate system parameters. In the case of asteroids, it is difficult to estimate higher order gravity terms, rotation and mass distribution. Results show that the direct adaptive controller is able to successfully track a desired hovering trajectory around the asteroid without assuming system parameters. It was seen that the controller was able to able handle unknown gravitational accelerations coupled with solar radiation pressure. In the future, this form of the controller combined with an existing controller can help to counter the effects of unknown models and external disturbances.

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