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Application of Direct Adaptive Control to Autonomous Satellite Docking

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This paper analyzes a Direct Adaptive Control algorithm applied to autonomous satellite docking problem. A linearized model of relative motion is used to describe the motion between satellites and an adaptive control algorithm is configured to regulate the position and velocity of the deputy to safely dock with the chief. A discretized control effort is applied to the system to minimize the amount of propellant required and the performance of the system is analyzed via simulation studies. An LQR controller is developed to generate an ideal trajectory for the adaptive system to follow in the presence of J2 disturbances. A linearized modified Hills-Clohessy-Wiltshire system with J 2 perturbation will be analyzed for the final version of this paper.

I. Introduction

An Adaptive Controller adapts its gains to closely resemble a desirable system or curb any variable uncertainties in the system. It can be classified into various categories based on their implementation or adaptation law they follow. Direct adaptive control schemes directly estimate the parameters used in the controller, whereas indirect methods estimate parameters that are then used to calculate the controller gains. An example of an adaptive system can be seen in fig.1. Open loop adaptive control takes advantage of a known relationship between some states and variables e.g. (Gain Scheduling). There are various control schemes that can be used to estimate the adaptive gains, some of the most frequently used methods are Model Reference Adaptive Control, Direct Adaptive Control, Neural Networks, Model Identification Adaptive controllers etc. In this research, direct adaptive control laws will be implemented.

Direct adaptive control [1] holds a distinct advantage over conventional controller designs in that it does not require a precise knowledge of the system model. For a linear model, in particular, direct adaptive control strategies can accommodate uncertainties in the system 'A' matrix. The sensor dynamics ('C') and the actuator dynamics ('B') are usually very well defined for aerospace grade sensors and actuators; hence there is usually a high confidence level associated with these values. The adaptive control scheme uses these B and C matrices to adapt to the control gains required by the system. Modeling of wind turbines, for example poses a challenge due to their complex machinery and their interaction with unpredictable conditions during operation, which poses difficulties in the development of a control system. Frost et al. extended the Direct Model Reference Adaptive approach for implementation in a utility-scale wind turbine for speed regulation [2], mitigating the challenges faced by other control schemes for lack of accurate dynamical modeling.

One of the most important functions of an autonomous system is to be able to follow a specified path, pertinent to the mission. Trajectory tracking is often accomplished using a combination of a waypoint path planner and a control system that is used to follow the computed way-points. Path planning is an extensively researched topic but is not pertinent to this investigation which focuses on the control schemes used to track these waypoints. The waypoints generated by path planners can be converted into 3-D trajectories. The controls effort required to achieve these maneuvers can then be used by the controller to follow the trajectory.

Any system in real world scenarios experiences disturbances, either external or mechanical (e.g vibrational, aeroelastic etc.). An ideal control system should be able to mitigate these unwanted inputs while continuing to perform

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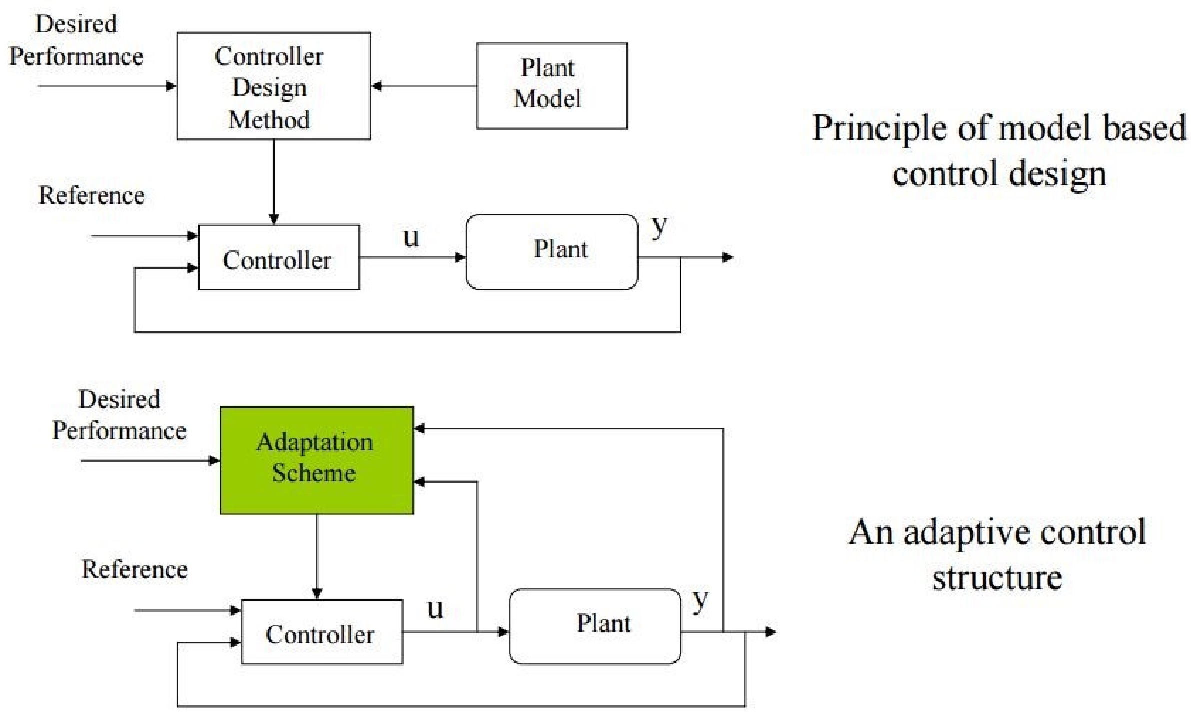

Fig. 1 Structure of an Adaptive System (Landau, Lonzano, M'Saad, and Karimi. 2011)

the tasks required. Disturbance Rejection becomes a challenge while utilizing classical control techniques such as PID controllers. This issue was highlighted by Han in [3], which also details the workings of Active Disturbance Rejection Control (ADRC) and its advantages in performance and practicality over PID.

II. Objective and Methodology

The objective of this paper is to analyze the performance of a Direct Adaptive Control scheme to a docking scenario of a deputy satellite into the chief satellite. The close proximity of the two satellites is used to describe a linearized equation of motion system.

A. Linear System

The relative motion between a 'Chief' and a 'Deputy' which are in close vicinity of each other is defined by the Clohessy-Wiltshire (CW) equations [4]. Relative accelerations in Cartesian coordinates are given by:

$$\begin{aligned} \ddot{x} - 2n\dot{y} - 3n^2x &= 0 \\ \ddot{y} + 2n\dot{x} &= 0 \\ \ddot{z} + n^2z &= 0 \end{aligned} \tag{1}$$

In equation 1, (x, y, z) represents the relative position of the two satellites in the orthogonal Cartesian coordinate system and n is the mean orbital rate. Some assumptions made for these equations to hold true are:

- Relative distance between the chief and deputy is much smaller than the orbit radius r.
- Relative orbit is assumed to be circular.

The state space system for the CW equations system is defined as follows:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{2}$$

where, $\underline{x} = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T$, $\underline{u} = \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix}^T$, and

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & n^2 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It is assumed that thrusters are available to control the deputy in any of the three directions and system states are observable.

B. Direct Adaptive Control

Adaptive Control uses gains that adapt based on changes in parameters or states of the system being controlled. Direct Adaptive Control is a direct feedback controller where the gains follow a highly nonlinear adaptation scheme depending on only the output of the linear system. In this work the adaptive controller is designed to enable the system to track trajectories from a linear reference model $\dot{x_r} = Ax_r + Bu_r$, $y = Cx_r$. The adaptation laws used in this research are derivatives of the schemes developed by Fuentes et. al. [5]. The adaptive control law can be represented as:

$$\begin{aligned} &u = G_e e_y + G_m x_r + G_r u_r + G_D z_D \\ &\dot{G}_e = -e_y e_y^T \sigma_e; \sigma_e > 0 \\ &\dot{G}_r = -e_y {x_r}^T \sigma_u; \sigma_u > 0 \\ &\dot{G}_u = -e_y u_r^T \sigma_r; \sigma_r > 0 \\ &\dot{G}_D = -e_y z_D^T \sigma_D; \sigma_D > 0 \end{aligned} \tag{3}$$

It should be noted that this adaptive controller requires that the number of inputs and outputs are the same (m = p). The reference state $x_r \epsilon \mathbb{R}^n$ and input $u_r \epsilon \mathbb{R}^m$ are the states and control inputs that correspond to the database of system trajectories that were generated using the linearized system models. $z_d \epsilon \mathbb{R}^d$ is the disturbance state and $e_y = y - y_r \epsilon \mathbb{R}^{mxd}$ is the output tracking error. The control structure includes four adaptive gains, $G_e \epsilon \mathbb{R}^{mxp}$, $G_r \epsilon \mathbb{R}^{mxn}$, $G_u \epsilon \mathbb{R}^{mxm}$, and $G_d \epsilon \mathbb{R}^{mxd}$, which are updated based on the nonlinear adaptation laws in Eq. 3. The adaptation law for each gain matrix is driven by the output tracking error e_y and include positive definite matrices $\sigma_e \epsilon \mathbb{R}^{pxp}$,$\sigma_r \epsilon \mathbb{R}^{nxn}$, $\sigma_u \epsilon \mathbb{R}^{mxm}$, and $\sigma_d \epsilon \mathbb{R}^{dxd}$. This adaptive control architecture, which is summarized in Figure 2, does not require a model of the system to be controlled, although a model is employed to generate the reference trajectories. Therefore, the adaptive controller can be applied with little knowledge of the system or for systems that are modeled with a significant degree of error.

The stability of the adaptive controller requires several key properties regarding the system model. Using theoretical results provided by Balas and Fuentes [6], the output and the adaptive gains are guaranteed to be bounded if the state-space system is Almost Strictly Positive Real (ASPR). This property is satisfied if there exists an output feedback gain matrix K such that the closed-loop system transfer function $\bar{P}(s) = C(sI - \bar{A})^{-1}B$, where $\bar{A} = A - BK$ is Strictly Positive Real (SPR) [7]. The ASPR property requires that the open-loop system transfer function $P(s) = C(sI - A)^{-1}B$ is minimum phase (i.e., all transmission zeros are stable), and it also requires that the matrix CB is positive definite (CB>0). Furthermore, the Robust Stabilization Theorem from Balas and Frost [8] can be used to prove that if the output tracking error is asymptotically stable provided that, in addition to the ASPR property, the following conditions hold:

- The disturbances can be described in terms of a set of bounded basis functions $\{\phi_i\}_{i=1}^d$
- The transmission zeros of the state-space model (A,B,C) must not coincide with the eigenvalues of the reference state-space model A_{ref} and the disturbance state-space model F. That is, it is required that $Z(A, B, C) \bigcap (\sigma(A_{ref}) \bigcup \sigma(F)) = \emptyset$.

The second condition is required in order to guarantee that there exists a unique solution to the matching conditions [9]. The existence of a unique solution to the matching conditions (or equivalently the existence of unique ideal trajectories) is required to guarantee the asymptotic stability of the adaptive controller. It should be noted that, in contrast to a Disturbance Accommodating Controller [10], the adaptive controller does not require the explicit solution of the matching conditions, merely the existence of a unique solution.

One of the main assumptions for implementation of an adaptive system is the realization of minimum phase. Relative motion of satellites, however, constitutes a non-minimum phase systems (i.e. they have neutrally stable transmission

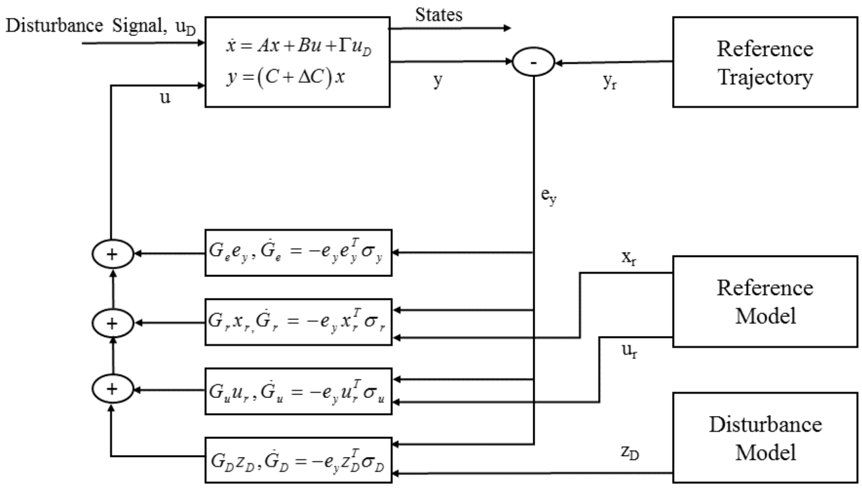

Fig. 2 Adaptive Control System Architecture

zeros). To mitigate this problem, Balas and Frost [11] developed a systematic approach to stabilize the unstable transmission zeros of a non-minimum phase system known as Sensor Blending.

In order to address non-minimum phase systems, the sensor blending process entails generating a new system output $y_\Delta \in \mathbb{R}^p$ corresponding to a linear combination of the original outputs $y \in \mathbb{R}^p$ (i.e., the sensor measurements):

$$y_\Delta = C_\Delta x = (C + \Delta C)x \tag{4}$$

The blended output is designed so that the resulting transfer function $P(s) = C_\Delta(sI - A)^{-1}$ is minimum phase, making it possible to guarantee the stability of the adaptive control system.

The sensor blending process entails first transforming the linear time-invariant system models into normal form. [6] showed that if CB is non-singular, then there exists a projection $P_1 = B(CB)^{-1}C$ onto the range of B along the null space of C. A projection $P_2 = (I - P_1)$ can then be defined as the complement of the projection P_1. The transformation to the normal form is then defined in terms of the non-singular matrix W:

$$z = \begin{bmatrix} y \\ z_2 \end{bmatrix} = Wx, W = \begin{bmatrix} C \\ {W_2}^T P_2 \end{bmatrix} \tag{5}$$

where $W_2 = {Q_2}^T P_2$ and the columns of Q_2 form a basis for the null space of C. The normal form is then defined as

$$\dot{z} = \bar{A}z + \bar{B}u, \;\; y = \bar{C}z \tag{6}$$

where the transformed system matrices are given by

$$\begin{aligned} \bar{A} &= WAW^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{21} \end{bmatrix} \\ \bar{B} &= WB = \begin{bmatrix} CB \\ 0 \end{bmatrix} \\ \bar{C} &= CW^{-1} = \begin{bmatrix} I_p & 0 \end{bmatrix} \end{aligned} \tag{7}$$

The state equation in normal form can then be written as

$$\begin{aligned} \dot{y} &= \bar{A}_{11}y + \bar{A}_{12}z_2 + CBu \\ \dot{z}_2 &= \bar{A}_{21}y + \bar{A}_{22}z_2 \end{aligned} \tag{8}$$

Eq. 8 represents the zero dynamics of the LTI system, which are invariant under transformation. The transmission zeros $Z(A, B, C)$ of an LTI system are given by [12]:

$$Z(A, B, C) = \left\{ \lambda \mid H(\lambda) \equiv \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} is\ singular \right. \tag{9}$$

The transmission zeros correspond to the eigenvalues of the sub matrix $\bar{A}_{22}$ in the normal form [13]. That is, $Z(A, B, C) = \sigma(\bar{A}_{22})$. It should be emphasized that the transmission zeros cannot be altered using output feedback. Therefore, [11] showed that, for cases in which there are unstable or neutrally stable transmission zeros, a blended output $y_\Delta = C_\Delta x = (C + \Delta C)x$ can be derived that yields stable transmission zeros of the form:

$$Z(A, B, C) = \left\{ \lambda \mid H(\lambda) \equiv \begin{bmatrix} A - \lambda I & B \\ C + \Delta C & 0 \end{bmatrix} = H(\lambda) + \begin{bmatrix} 0 & 0 \\ \Delta C & 0 \end{bmatrix} \right. \tag{10}$$

This is accomplished by first decomposing the second equation in the normal form given in Eq. 8 into stable and unstable subsystems:

$$\begin{bmatrix} \dot{z}_2^s \\ \dot{z}_2^u \end{bmatrix} = \begin{bmatrix} \bar{A}_{22}^s & 0 \\ 0 & \bar{A}_{22}^u \end{bmatrix} + \begin{bmatrix} \bar{A}_{21}^s \\ \bar{A}_{21}^u \end{bmatrix} y \tag{11}$$

where $\sigma(\bar{A}_{22}) = \sigma(\bar{A}_{22}^s) \cup \sigma(\bar{A}_{22}^u)$ with $\sigma(\bar{A}_{22}^s)$ denoting the stable eigenvalues of $\bar{A}_{22}$ (i.e, the stable transmission zeros) and $\sigma(\bar{A}_{22}^u)$ representing the stable eigenvalues (i.e, the unstable transmission zeros). In Theorem 6 from [11], the authors show that if CB is nonsingular and the pair $(\bar{A}_{22}^u, \bar{A}_{21}^u)$ is controllable, then there exists a ΔC such that for all $Re(\lambda) \geq 0$, $H_\Delta(\lambda)$ as defined in Eq. 10, is invertible. In other words,under these conditions, a blended output $y_\Delta = C_\Delta x = (C + \Delta C)x$ exists such that the resulting transmission zeros $Z(A, B, C + \Delta C)$ are all stable. A blended output can then be derived by computing a feedback matrix K_Δ such that $\sigma(\bar{A}_{22}^u - \bar{A}_{21}^u K_\Delta)$ is stable. The blending matrix ΔC can then be computed as $\Delta C = K_\Delta {Q_2}^T P_2$. It should be noted that the blending process does not alter CB (i.e., $C_\Delta B \equiv (C + \Delta C)B = CB$).

III. Implementation and Results

A MATLAB/SIMULINK simulation was built to simulate the relative motion and docking scenario. The linear system described in the methodology is used along with the adaptive controller structure mentioned in equation 3. The system is assumed to be under J2 Oblateness perturbation, which is modeled as a disturbance and injected into the system.

The system is assumed to be under J2 Oblateness perturbation, which is modeled as a disturbance and injected into the system. Schweighart [14] developed a linearized J2 model which simulates the effect of earth-oblateness, the linear system is given below:

$$\ddot{x} - 2nc\dot{y} - (5c^2 - 2)n^2 x = -3n^2 J_2 \frac{{R_e}^2}{r_{ref}} (\frac{1}{2} - \frac{3sin^2 i \cdot sin^2(nct)}{2} - \frac{1 + 3cos2i}{8}) \tag{12}$$

$$\ddot{y} + 2nc\dot{x} = -3n^2 J_2 \frac{{R_e}^2}{r_{ref}} sin^2 i \cdot sin^2(nct) \cdot cos(nct) \tag{13}$$

$$\ddot{z} + (3c^2 - 2)n^2 z = -3n^2 J_2 \frac{{R_e}^2}{r_{ref}} sini \cdot cosi \cdot sin(nct) \tag{14}$$

where, n = mean orbital rate, s = $\sqrt{1+s}$, s = $3J_2 R_e \frac{1+3cos(2i)}{8r_{ref}}$, t = time, and i = angle of incidence. The R.H.S of 12-14 forms the disturbance model which is added to the linear system (L.H.S).

The Adaptive system is configured to regulate the states of this linear system. Since, the relative motion described by the C-W equations constitutes a non-minimum phase system (transmission zeros are neutrally stable) a sensor blending approach was taken. Bass-Gura algorithm [15] was used to place the three neutrally stable zeros to $[-1, -1.5, and, -2]$ respectively. The initialization of the deputy is done at 0.5,1,0.1 km in the x, y and z direction, while the initial velocities are chosen to be 0.1, 0, 0.1 km/s respectively. The altitude of the chief with respect to the center of the earth is chosen to be 6776 km (Low-Earth Orbit).

A. Direct Adaptive Controller Results

For the first case, a direct adaptive controller was used to regulate the linear system states to zero (signifying docking), in presence of the J2 disturbance. Representative results are presented below.

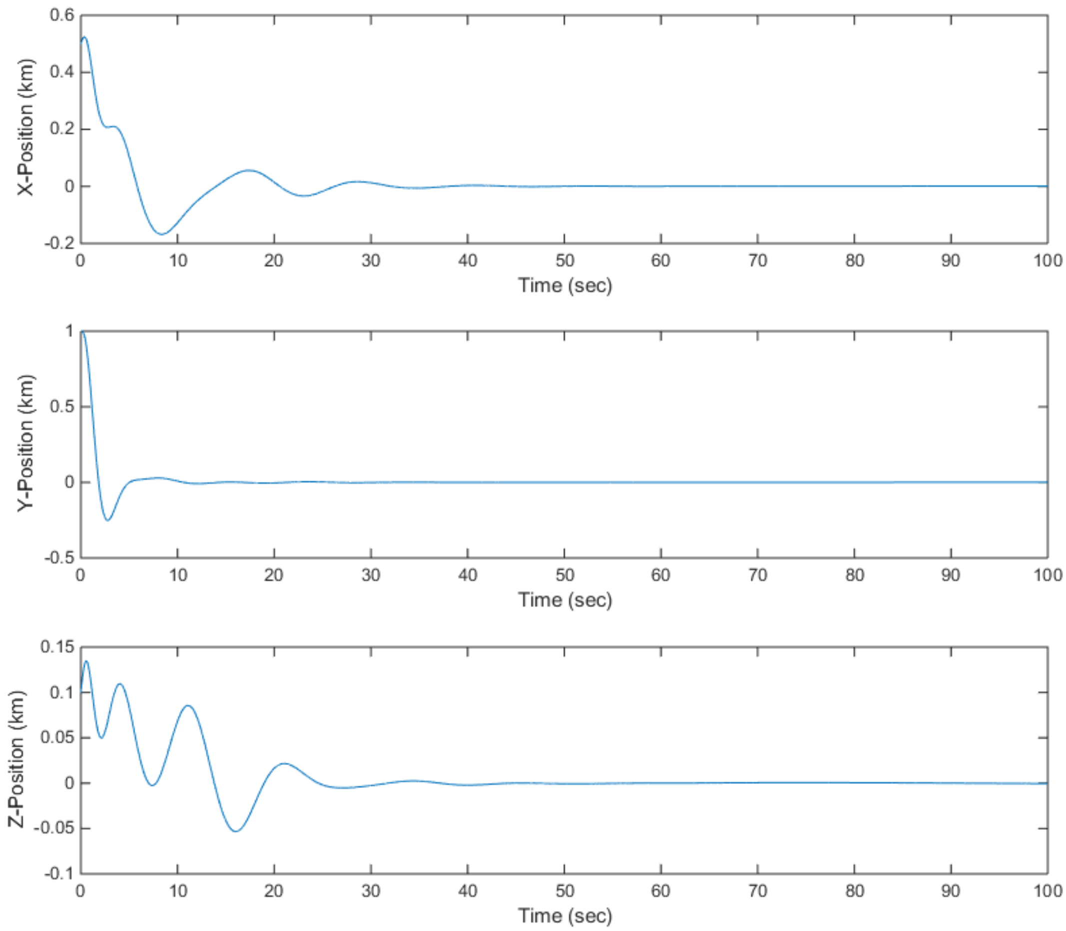

Fig. 3 Relative Position w.r.t Chief while using Direct Adaptive Controller for Docking

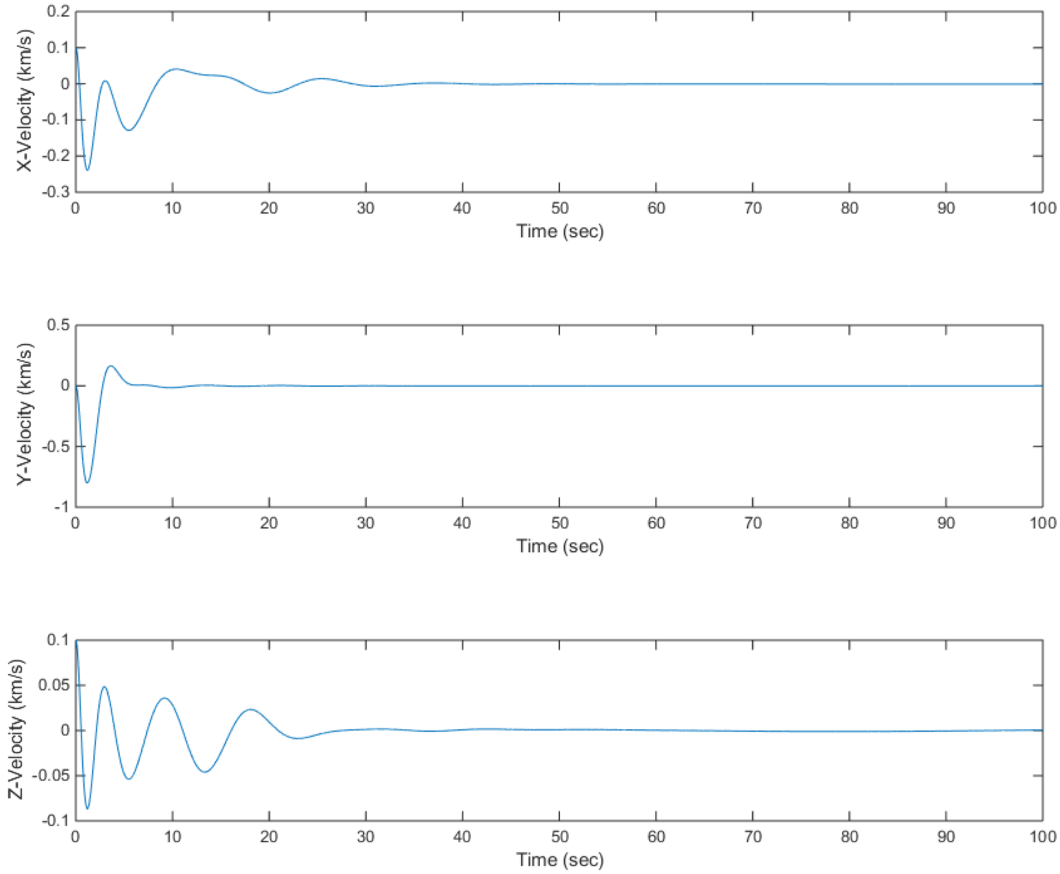

Fig. 4 Relative Velocity w.r.t Chief while using Direct Adaptive Controller for Docking

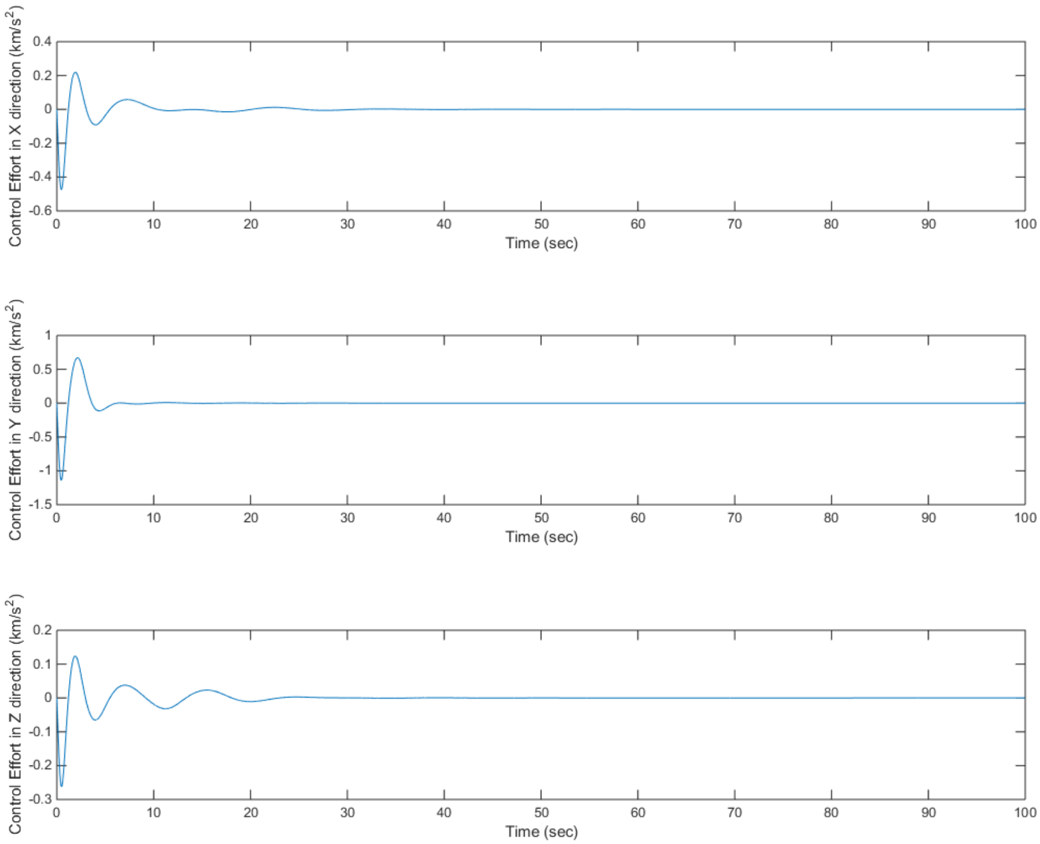

Fig. 5 Control Effort required while using Direct Adaptive Controller for Docking

As can be inferred from Figures 3-5, the adaptive system is able to achieve the goal of regulating the states (position and velocity) of the deputy to zero, which implies docking. This maneuver is performed in a time span of 100 seconds with an steady state error of about 1 meter in position and 10cm/s in velocity. However, this only serves as a proof of concept for the working of an adaptive control scheme in a docking scenario as there is active control and the control efforts are large, making it rather impractical for spacecraft docking application.

B. Optimal Trajectory Following using Direct Adaptive Controller

Since, the direct adaptive control scheme produced control efforts that were large, an infinite horizon optimal regulator (LQR) was used to obtain a reference trajectory for the adaptive controller to follow. This allows for minimizing a cost function while penalizing control effort (using a high value of $R \approx 10^6$). The gain obtained from this minimization is then used in a closed loop system to generate a trajectory for the adaptive system to follow. The results of this simulation are presented below.

As can be observed in Figures 6 and 7, the adaptive controller successfully tracks the reference optimal trajectories, while rejecting disturbance/perturbations. Figure 8 depicts the control effort required by the direct adaptive controller to follow this trajectory. Comparing it to figure 5 where only the direct adaptive controller was used, there is an order of magnitude decrease in the control effort. This is an important factor to consider while discussing any spacecraft controls problem as the propellant/control effort is the limiting factor in these scenarios. This system can be used as a fail-safe method in dire conditions where there are unknown disturbances/modeling errors that the system is experiencing that are impossible to model.

J2 perturbations do not play a major role in the HCW equations when studying the docking problem, this is because of the close proximity of the two satellites, and the short duration of time the objects are under the effect of these

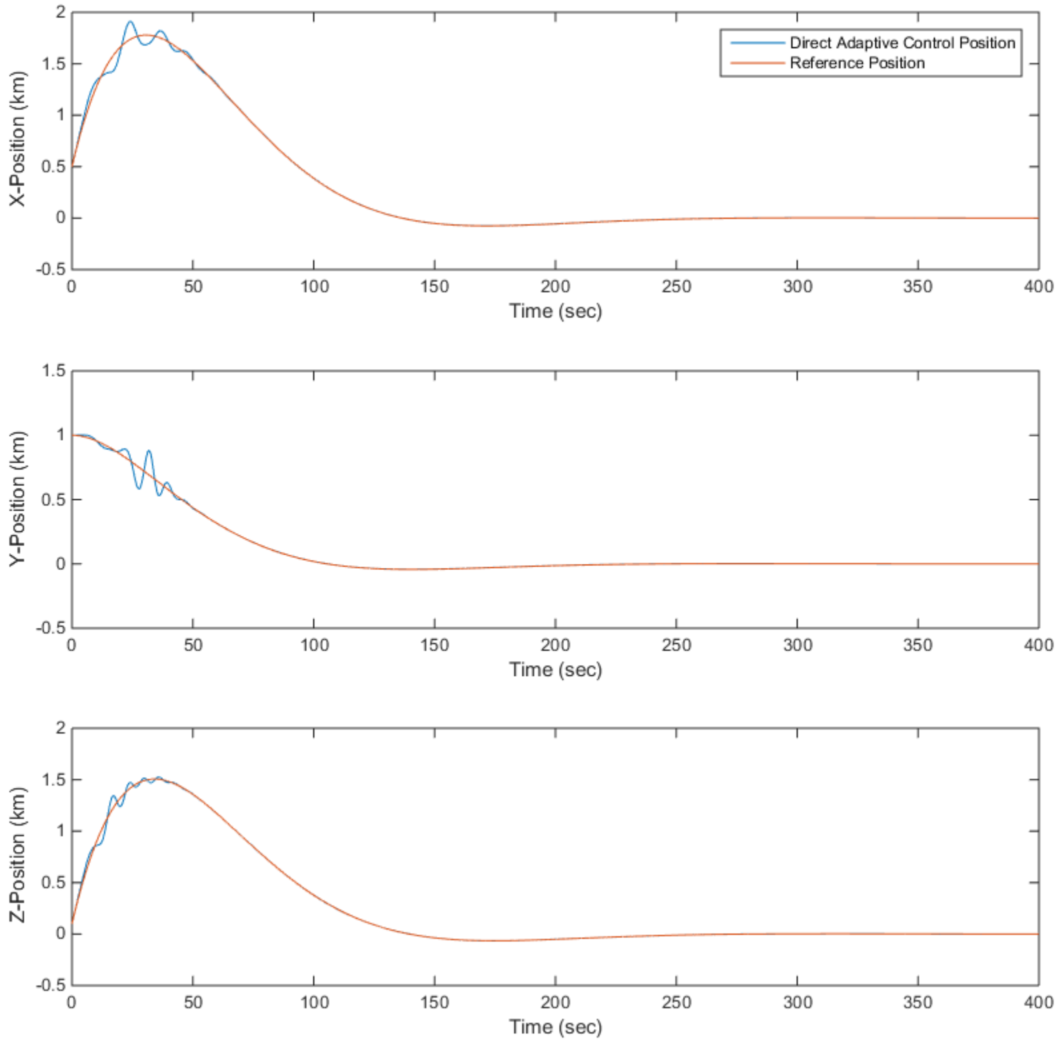

Fig. 6 Relative Position w.r.t Chief while using Optimal Trajectory Tracking for Docking

disturbances. This is evident from the magnitude of disturbance, which is around 10^{-7}. This can be observed in the figure 9 below:

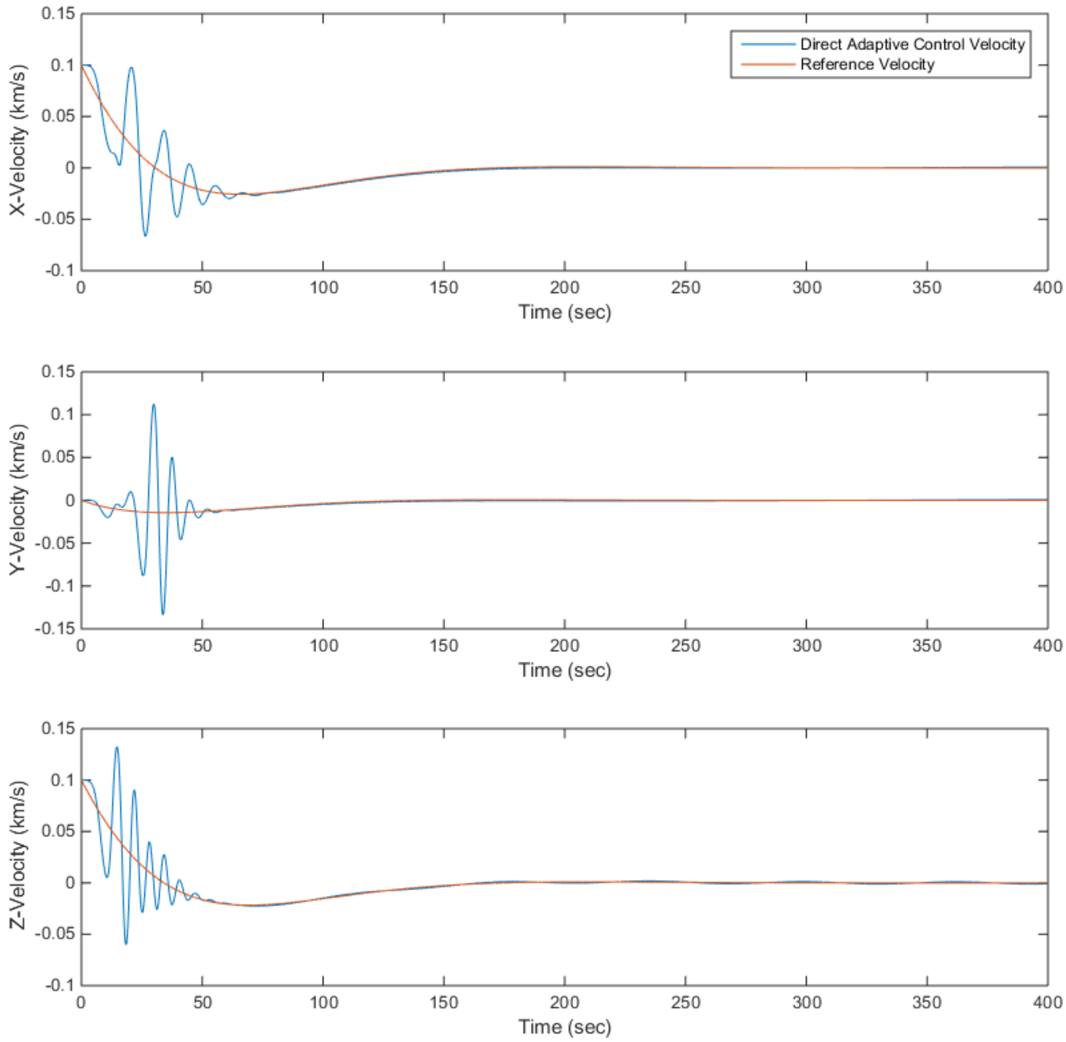

Fig. 7 Relative Velocity w.r.t Chief while using Optimal Trajectory Tracking for Docking

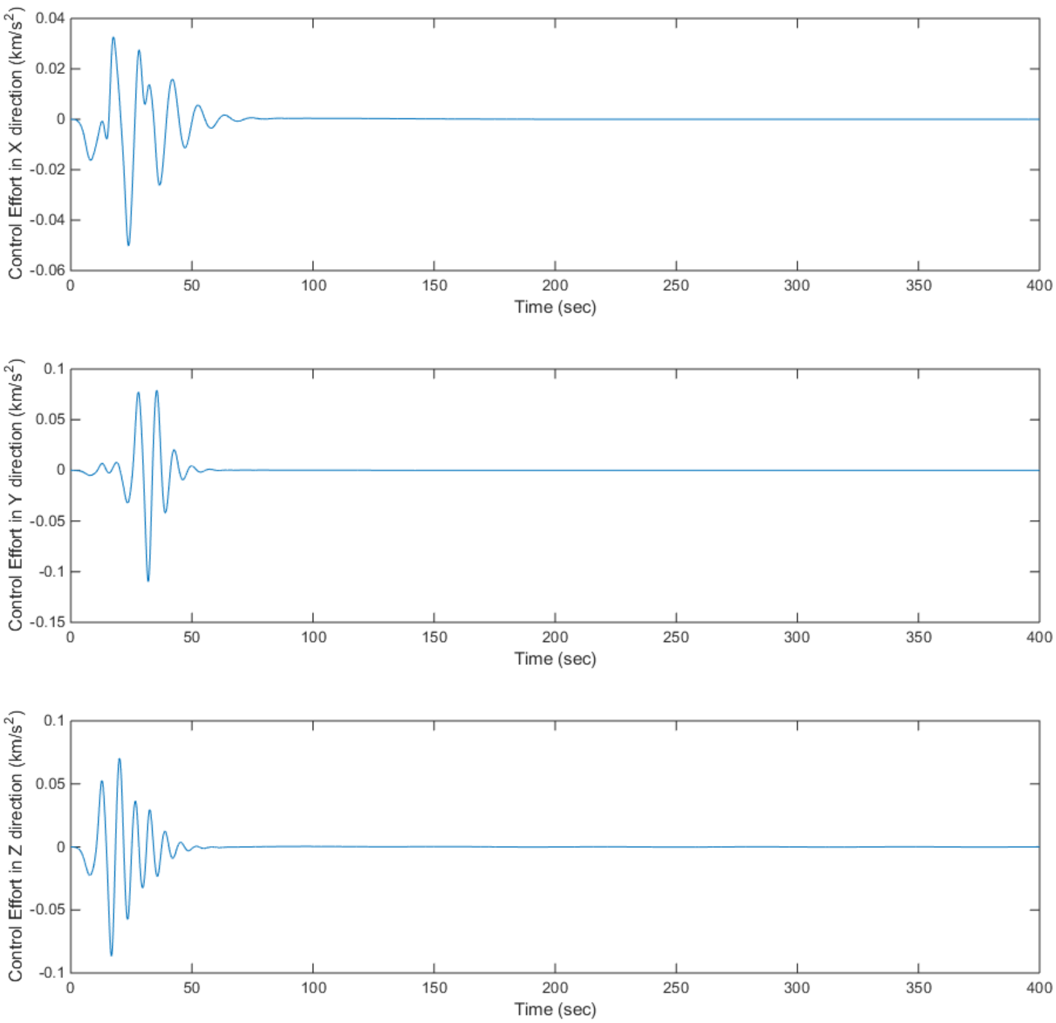

Fig. 8 Control Effort required while using Optimal Trajectory Tracking for Docking

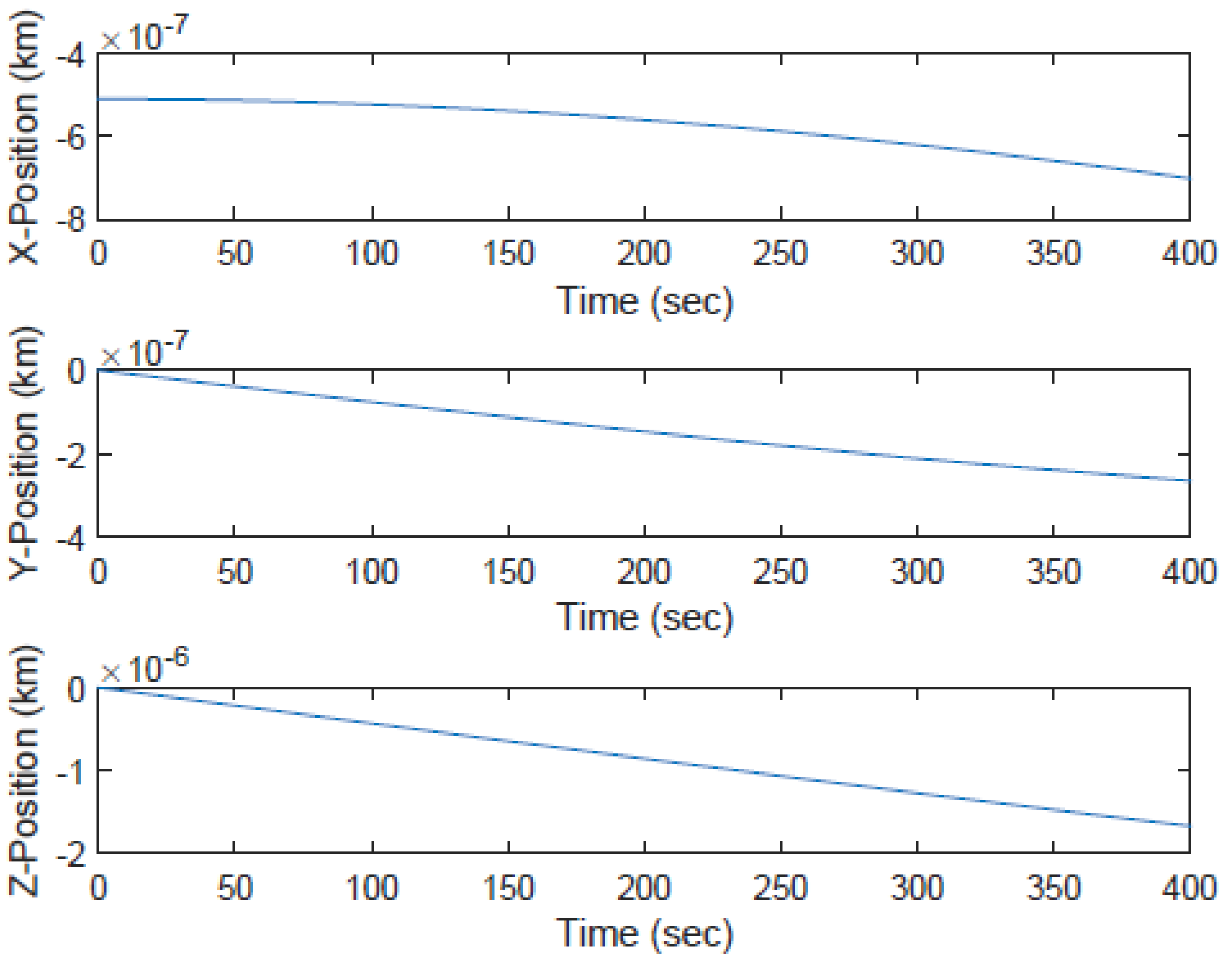

Fig. 9 J2 Perturbation Magnitude over a period of 400 seconds

IV. Conclusion

In this paper, Clohessy-Wiltshire equations were used to simulate the relative motion between two satellites, and an adaptive control solution to the docking problem was developed. Direct Adaptive control scheme was used as the primary control system for this problem. The control scheme was adapted to this linear system by using sensor blending to satisfy the assumptions required to prove stability of the adaptive controller. Two methods were used to regulate the states of the deputy satellite and allow it to dock with the chief. The first method comprised of only using the direct adaptive scheme. The controller is able to regulate the system while mitigating the J2-Oblateness perturbations but the resulting control effort observed was large. The second scenario consisted of developing a LQR controller to generate reference trajectories that minimize the control effort; this resulted in reduction of control efforts by an order of magnitude.

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